Firm Subsidies and Entrepreneurship in A Production Network

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Abstract

I develop a quantitative framework to study the effect of a direct subsidy program for firms in an environment featuring a production network and entrepreneurship. The combination of a network structure and sectoral financial constraints can propagate shocks from upstream to downstream and affect the transition path of the aggregate economy. Quantitatively, the recent firm subsidy program during COVID-19 increased aggregate consumption and induced excessive firm creation. Redesigning the distribution of funds across sectors could lead to improvements in aggregate outcomes.

1 Introduction

Direct subsidies have become an increasingly common policy tool during economic downturns, aimed at providing liquidity support to firms under financial stress. A recent example is the Coronavirus Aid, Relief, and Economic Security Act (CARES Act). Of the $2.2 trillion stimulus package, $500 billion was allocated as loans to corporations, and $953 billion was provided as forgivable loans to businesses under the Paycheck Protection Program (PPP). The rationale behind these programs is to offer credit relief to prevent widespread business closures. On the other hand, modern production economy features heterogeneous sectors that are interconnected. During the COVID-19 pandemic, the impact was uneven across sectors, with contact-intensive industries being disproportionately affected. Despite this, such programs seldom consider sectoral linkages. This raises a key question: should we redesign firm subsidy programs to account for the underlying production network of the U.S. economy?

This paper presents a quantitative framework that can be used to evaluate direct subsidy programs, in terms of both aggregate and sectoral outcomes. I extend a production network model to include financial frictions, capital accumulation and entrepreneurship. These features allow me to study the transitions of aggregate outcomes and sectoral firm dynamics in response to shocks. I estimate that with a total size of around 4% GDP of 2019, the PPP decreased the cumulative impact of COVID-19 shock on aggregate consumption from -22% to -12% over a 5-year period. Redesigning the distribution of the PPP yields an 1 percentage point gain in aggregate consumption. Additionally, it attenuates the surge in firm entry among downstream sectors observed in the data.

Firm subsidy programs are designed to mitigate production distortions caused by financial frictions. This issue is especially pressing during economic crises, when financially strained firms face significant profit losses and may ultimately shut down. Therefore, firm dynamics are often a primary metric for evaluating these programs. Furthermore, the modern production economy exhibits considerable heterogeneity and complex linkages between sectors. I capture these stylized facts by introducing intertemporal decisions on the household side in an otherwise static production network model.

Firm's entry and exit decision is modeled as occupational choice in my model in the spirit of Buera, Kaboski, and Shin (2011). Each period, households can decide to either work and earn a wage or run their own firm. The combination of financial frictions and the production network determines the pre-shock equilibrium. Firms rent capital to operate but are subject to financial constraints. As a result, entrepreneurial decisions are distorted, as productive but financially constrained entrepreneurs cannot operate at a sufficient scale. Hence, the equilibrium distribution skews toward wealthier but less productive entrepreneurs. Within a production network, such distortions in one sector can affect others through shifts in intermediate prices. Changes in production costs affect the scales of operations in the downstream sectors, altering their equilibrium distributions. Moreover, as the level of financial constraint rises in the economy, aggregate savings exceeds the level in a perfect-credit benchmark, leading to lower interest rates and further distorting entrepreneurial selection across sectors.

When faced with a negative shock, businesses' (entrepreneurs') profits decline, leading them to consume their wealth due to consumption smoothing. If they are financially constrained, this forces them to reduce their scale of operation, which further lowers their profits. Entrepreneurs close their businesses when profits fall below the market wage, the opportunity cost of not working for other firms. This alters the distribution of active businesses and affects sectoral productivity. These changes are propagated through intermediate prices within the production network. As a result, firm dynamics in one sector depend not only on the wealth distribution of its own entrepreneurs but also on those in upstream and downstream sectors. As entrepreneurs gradually re-accumulate their wealth, the economy transitions back to steady state.

In my framework, financial frictions and production networks together shape both the initial responses and the subsequent transition paths in response to aggregate and sectoral shocks. Poor financial conditions in upstream sectors amplify price movements, which trigger larger responses and prolong transitions in downstream sectors. I call this the "transmission effect." This mechanism merges the "persistence of history" from Buera and Shin (2013) with the propagation effects studied in the network literature (e.g. Acemoglu et al., 2012). Differences in the pre-shock entrepreneur distributions have an ambiguous effect on firm dynamics. Persistent idiosyncratic productivity and financial constraints together create a "cleansing effect," where both productive but poor firms and wealthy unproductive firms exit the market, but at varying rates depending on the pre-shock entrepreneur distributions.

In my numerical exercise, I disentangle these forces by studying negative shocks of varying persistence. Transitory shocks primarily work via "transmission effect", as they have limited impact on the wealth distribution of entrepreneurs. In contrast, long-lasting shocks deplete more capital, making the changes in distribution more pronounced and significantly affecting firm dynamics. In an illustrative case, due to "cleansing effect", a downstream sector can experience more exits when the level of financial constraint in its upstream is loose, if the shock is persistent enough.

The production network also amplifies the response to aggregate productivity shocks. In an economy with connected downstream sectors, aggregate consumption declines more compared to an economy without these connections. When sectors are interconnected, aggregate shocks generate feedback effects through the production network, leading to a larger drop in downstream sectoral outputs. The "cleansing effect" mentioned above is also present in this case. The same persistence exercise shows that the first mechanism dominates when the shock is transitory, as there is little change in the distribution of entrepreneurs' wealth. As a result, there are more exits when the sectors are connected. However, with a shock that is persistent enough, this relationship can be reversed.

I structurally estimate this model using pre-COVID-19 firm-level data and the industrial input-output table in the U.S. To account for the heterogeneity within the production network, I estimate the production function non-parametrically for each sector. For parameters related to financial frictions and productivity processes, I use a simulated method of moments to match existing empirical evidences. Chaney, Sraer, and Thesmar (2012) use exogenous variations in real estate values to find casual estimates on the response in firms' investment to a relaxation on their borrowing constraints. I run the same specification in their paper with a simulated panel of firms and match the estimated coefficients by sectors. To address the differences between the variables in the empirical design and variables available in the model, I proxy firm investment by growth in capital inputs, and real estate value by entrepreneur's wealth. The productivity process is estimated to match the volatilities in sales at different horizons in the spirit of Midrigan and Xu (2014). The baseline estimation reveals considerable heterogeneity between sectors in terms of their steady-state distributions and responses to shocks.

During COVID-19, sectors were unevenly affected and exhibited varying recovery speeds and firm dynamics. I estimate a series of COVID-19 shocks that generate impulse response functions in the model similar to those observed in the data. The shocks are consist of a negative aggregate labor supply shock and sectoral demand shocks to capture the uneven impacts across sectors. These are then used as a basis to evaluate the performance of the Paycheck Protection Program (PPP). With a total size of $953 billion, equivalent to 4% of U.S. GDP in 2019, the PPP reduces the cumulative impact of the COVID-19 shock on aggregate consumption from -22% to -12% over a five-year horizon. Beyond aggregate outcomes, the PPP has only modest effects on sectoral output but heterogeneous impacts on sectoral firm dynamics. Notably, the PPP contributes to the surge in new business applications observed in the data (Fazio et al., 2021; Decker and Haltiwanger, 2023). Sectors such as Construction, Real Estate, and Education and Health experience significant increases in business formations compared to pre-COVID-19 levels.

Redesigning the PPP by altering the fund distribution while keeping its total size and rollout speed is able to achieve 1 percentage point increase in terms of aggregate consumption. Comparing to the original PPP, it assigns more funds to Food and Entertainment, which was most severely hit, and Construction, which serves as an important part of the production network. With this counterfactual policy, the surge in business creation seen in many sectors are attenuated. I interpret these results as evidence of inefficient allocation of funds and subsequent excessive firm creations.

The idea of "saving oneself out of financial constraints" originates from the entrepreneurship literature and is often applied in a development context, focusing on aggregate productivity. Buera, Kaboski, and Shin (2011) use the U.S. economy as a perfect-credit benchmark to explore the impact of varying levels of financial frictions on aggregate outcomes and selection into entrepreneurship. They find that financial frictions can explain much of the variation in sectoral productivity. Buera and Shin (2013) use a single-sector model to study the transition dynamics following reforms that eliminate existing exogenous distortions, such as taxes and industrial policies. In their quantitative analysis, financial frictions influence the speed of the transition to a new distortion-free steady state. Moll (2014) shows that the speed of transition also depends on the parameters of the idiosyncratic productivity process. Specifically, a more persistent stochastic process reduces the impact of financial frictions but leads to a slower transition. In my framework, the structure of production network also shapes the distributions of entrepreneurs at the steady-state. Depending on the nature of shocks, different production networks can lead to vastly different transition dynamics.

My results also contribute to the production network literature, particularly in relation to studies on misallocation within networks. Baqaee and Farhi (2019) and Baqaee and Farhi (2020) formulate and generalize the concept of misallocation within networks caused by distortions, such as markups. Liu (2019) takes a similar approach, discussing the role of industrial policies in addressing these distortions. In this strand of literature, the supply of production factors is assumed to be inelastic, so distortions affect aggregate outcomes solely through the misallocation of resources. Bigio and La'O (2020) introduce endogenous labor supply and show that even in extreme network structures where misallocation is impossible within the network, distortions can still create a wedge in labor supply. However, these frameworks are intratemporal in nature and not well-suited for analyzing transition dynamics. My framework introduces two intertemporal components in the supply of production factors: capital and the set of firms. Negative shocks can have long-lasting effects due to the depletion of capital (wealth). Although entry and exit decisions are intratemporal, they depend on the slow-moving wealth distribution because of financial constraints.

My paper contributes to the growing literature on dynamic production networks. Liu and Tsyvinski (2024) generalize the setup in Acemoglu et al. (2012) by incorporating dynamic adjustment costs in input-output linkages, showing that these linkages take time to recover after a negative productivity shock. My model captures a similar idea via a different approach: while the production network is static, sectoral productivities adjust gradually due to financial frictions and firm dynamics. In other words, my model introduces dynamics on the household side, whereas Liu and Tsyvinski (2024) introduce them on the production side. Notably, Liu and Tsyvinski (2024) find that sectors within the network respond differently to transitory and persistent shocks depending on the network's structure. This echos my findings with respect to the "cleansing effect". Lastly, while Liu and Tsyvinski (2024) focus more on analytical exploration, I trade tractability for a realistic representation of sectoral firm dynamics.

The quantitative exercise regarding the PPP contributes to the broader literature on the economic impact of COVID-19 and the corresponding fiscal policies. Among others, Faria-e-Castro (2021) uses a calibrated DSGE model with two sectors to studies the effect of various fiscal policies. My framework differs in two key ways. First, firm entry and exit decision are modeled as occupational choice, subject to the financial constraint. Thus, firm dynamics depend on the wealth distribution, which changes over time via households' consumption-saving behaviors. In this case, firm subsidy improves the wealth condition of entrepreneurs and has long-lasting effects. Second, the extensive production network in my framework allows for heterogeneous responses and feedback between sectors, which is absent in a two-sector model. Guerrieri et al. (2022) use a similar setup but focus more on analytical exploration. They emphasize that in a multi-sector economy with incomplete markets, supply shocks can lead to demand shortages through general equilibrium effects. This provides theoretical support for the COVID-19 quantitative exercises in my paper.

This paper also connects to the broad literature on heterogeneous agent (HA) models (Aiyagari, 1994; Moll, 2014; Kaplan, Moll, and Violante, 2018; Ahn et al., 2018) and heterogeneous firm models (Ottonello and Winberry, 2020; vom Lehn and Winberry, 2021). Similar to those models, the equilibrium distribution of entrepreneurs determines the response to shocks on impact and on the transition path. On the solution technique, I utilize the continuous-time finite difference method developed by Achdou et al. (2022) to solve the equilibrium efficiently. In my quantitative exercise, solving the transition dynamics become numerically taxing as the number of sectors increases. To combat this, I use the sequence-space Jacobian method by Auclert et al. (2021). The key idea behind their method is that sequence-space Jacobians, the derivatives of perfect-foresight equilibrium mappings between aggregate sequences around the steady state, are sufficient statistics that summarize all the relevant information about heterogeneity in order to determine general equilibrium dynamics, to first order with respect to aggregate shocks.

The roadmap of the rest of the paper is as follows: Section 2 presents the model and uses numerical exercises to demonstrate the key mechanisms. Section 3 calibrates the model to pre-COVID-19 US data. Section 4 use the calibrated model to study the aggregate and sectoral responses during the COVID-19 episode, and evaluate the effect of the PPP. Then I explore counterfactual redesign of the PPP under different criterions. Section 5 concludes.

2 Model

In this section, I present an economy with an arbitrary production network and sector-specific borrowing constraints. Households make occupational choice frictionlessly each period as well as production decisions. Households also determine the aggregate capital supply via savings. Financial frictions have both intratemporal and intertemporal impacts. Intratemporally, financial frictions affect resource allocations and firm entry and exit margins within the network. Intertemporally, financial frictions enter household's saving decision and distort aggregate capital accumulation.

Time, denoted by t, is infinite and continuous. There is a continuum of households, indexed by i, with measure $\mathcal{S}$. The households are ex-ante heterogeneous in terms of the technology they are exogenously born with. The households are equally divided into $\mathcal{S}$ sectors, with corresponding production functions F_s. Denote the set of households born with F_s by $\mathcal{I}_s$. Each instant, the households decide how much to consume and their occupation, subject to idiosyncratic productivity $z_{i,t}$, which follows a Ornstein–Uhlenbeck process.

The optimization problem of household i with sector s technology at t is

$$\begin{aligned}
\max_{\{c_{i,\tau}\}} \mathbb{E}_t &\left[\int_t^\infty e^{-\rho(\tau-t)} u\left(c_{i,\tau}\right) d\tau\right] \\
da_{i,t} &= \left[y_{i,t}\left(z_{i,t}, a_{i,t} \middle| \left\{P_{h,t}\right\}_{h\in\mathcal{S}}, r_t^k, w_t\right) + r_t^k a_{i,t} - c_{i,t}\right] dt \\
y_{i,t}\left(\cdot\right) &= \max\left\{w_t, \pi_{i,t}\left(z_{i,t}, a_{i,t} \middle| \left\{P_{h,t}\right\}, r_t^k, w_t\right)\right\} \\
d\log\left(z_{i,t}\right) &= \mu\left(\bar{z} - z_{i,t}\right) dt + \sigma dW_{i,t} \\
i &\in \mathcal{I}_s,\ \forall s \in \mathcal{S}
\end{aligned}$$

where $a_{i,t}$ is household's wealth, $\{P_{h,t}\}_{h\in\mathcal{S}}$ are prices of sectoral goods, r_t^k is the interest rate, w_t is the market wage, $y_{i,t}(\cdot)$ is the income that comes from either wage or entrepreneurial profit $\pi_{i,t}$, which comes from production operation. μ is the level of mean reversion of the stochastic process, $\bar{z}$ is mean, σ is the volatility. I assume all households share the same specification of $z_{i,t}$. If the household chooses to be a worker, it supplies one unit of labor inelastically, and earns w. If it chooses to be an entrepreneur, the household exits the labor force temporarily and hires other workers for operation. The entrepreneurial decision is made based on

$$w_t \lesseqgtr \pi_{i,t}\left(z_{i,t}, a_{i,s,t} \middle| \left\{P_{h,t}\right\}_{h\in\mathcal{S}}, r_t^k, w_t\right),\ i \in \mathcal{I}_s,\ \forall s \in \mathcal{S}.$$

Each entrepreneur in sector s produces the same sector s product. The production process uses capital k, labor l, and intermediary inputs from all sectors $\{x_h\}_{h\in\mathcal{S}}$. The production function of a entrepreneur i in sector s at time t is:

$$\begin{aligned}
q_{i,t} &= z_{i,t} F_s\left(k_{i,t}, l_{i,t}, x_{i,t}\right) = z_{i,t} k_{i,t}^{\alpha_s} l_{i,t}^{\beta_s} x_{i,t}^{\gamma_s}, \quad \alpha_s + \beta_s + \gamma_s < 1, \forall i, s \\
x_{i,t} &= G_s\left(\mathbf{x}_{i,t}\right) = \Pi_{h\in S} x_{i,h,t}^{g_{s,h}}, \quad \sum_{h\in S} g_{s,h} = 1,\ i \in \mathcal{I}_s,\ \forall s \in \mathcal{S}
\end{aligned}$$

$x_{i,h,t}$ is the sector h output used by entrepreneur i as intermediate input. $\mathbf{G}$, where $\mathbf{G}_{s,h} = g_{s,h}$ captures the network structure of the production economy. Entrepreneur earns profit:

$$\pi_{i,t} = P_{s,t} q_{i,t} - w_t l_{i,t} - r_t' k_{i,t} - \sum_h P_{h,t} x_{i,h,t},\ i \in \mathcal{I}_s,\ \forall s \in \mathcal{S}. \tag{2.1}$$

$P_{s,t}$ is the price of sector s good. r_t' is the rental rate of capital. The entrepreneur rents capital to operate subject to a borrowing limit proportional to its own wealth:

$$k_{i,t} \leq \theta_s a_{i,t},\ i \in \mathcal{I}_s,\ \forall s \in \mathcal{S} \tag{2.2}$$

This borrowing limit is the only source of friction in this model. To account for the heterogeneity between sectors, I allow θ_s to be sector-specific.

There is a competitive finance sector, outside of the production sector space $\mathcal{S}$, that facilitates saving and lending. In particular, a representative intermediary takes saving from the households at rate r_t and lending to entrepreneurs at rate r'_t. After the production phase, the intermediary uses the same saving technology as the household to convert final consumption goods into new capital to make up for the depreciation, δ, before returning it to the households. In equilibrium, perfect competition implies $r'_t = r_t + \delta$. This is the effective borrowing rate faced by the entrepreneurs.

There is a representative final consumption good producer that aggregate sectoral goods into final consumption good via a Cobb-Douglas technology:

$$C_t = \prod_s C_{s,t}^{\nu_s}, \;\; \sum_s \nu_s = 1$$
$$\pi_t = P_t C_t - \sum_s P_{s,t} C_{s,t}$$

Because this representative firm has constant return-to-scale (CRS) technology and behave competitively, it earns zero profit in equilibrium.

The problem of the household can be expressed recursively. Since the production decision is intratemporal in nature, the state variables of the household are, wealth, a, idiosyncratic productivity, z, and aggregate prices, $\left\{r\left(t\right), w\left(t\right), \left\{P_s\left(t\right)\right\}_{s\in\mathcal{S}}\right\}$. The Hamilton-Bellman-Jacobian (HJB) equation of a household i in sector s is

$$\begin{aligned}\rho v\left(a_{i,t}, z_{i,t}, t\right) &= \max_{c_{i,t}} u\left(c_{i,t}\right) + \partial_a v\left(a_{i,t}, z_{i,t}, t\right)\left[y\left(z_{i,t}, a_{i,t}|t\right) + r\left(t\right)a_{i,t} - c_{i,t}\right] \\ &+ \partial_z v\left(a_{i,t}, z_{i,t}, t\right)\mu\left(z_{i,t}\right) + \frac{1}{2}\partial_{zz} v\left(a_{i,t}, z_{i,t}, t\right)\sigma^2\left(z_{i,t}\right) + \partial_t v\left(a_{i,t}, z_{i,t}, t\right), \; i \in \mathcal{I}_s, \; \forall s \in \mathcal{S} \quad (2.3)\end{aligned}$$

The dynamic of the distribution can be analytically expressed via the Kolmogorov Forward Equation (KFE):

$$\begin{aligned}\partial_t g_s\left(a, z, t\right) &= -\partial_a\left[s\left(a, z, t\right)g_s\left(a, z, t\right)\right] - \partial_z\left[\mu\left(z\right)g_s\left(a, z, t\right)\right] + \frac{1}{2}\partial_{zz}\left[\sigma^2\left(z\right)g_s\left(a, z, t\right)\right] \quad (2.4) \\ s\left(a_{i,t}, z_{i,t}, t\right) &= y\left(z_{i,t}, a_{i,t}|r_t^k, w_t\right) + r\left(t\right)a_{i,t} - c\left(a_{i,t}, z_{i,t}, t\right), \; i \in \mathcal{I}_s, \; \forall s \in \mathcal{S}.\end{aligned}$$

g_s is the distribution of households in sector s. Each sector follows its own KFE and the total mass of each sector stays constant.

Definition 1. Stationary Equilibrium

A stationary equilibrium of this model is a set of aggregate prices $\left\{\{P_{s,t}\}_{s\in\mathcal{S}}, r_t, w_t\right\}_{t\in[0,\infty)}$, and households' actions $\{C_{i,t}, e_{i,t}, k_{i,t}, l_{i,t}, x_{i,h,t}\}_{i\in\mathcal{I}_s,\forall s,h\in\mathcal{S},t\in[0,\infty)}$, final consumption good aggregator's action $\left\{\{C_{s,t}\}_{s\in\mathcal{S}}\right\}_{t\in[0,\infty)}$ such that given an initial distribution $\{g_s(a,z,0)\}_{s\in\mathcal{S}}$ and the stochastic process of idiosyncratic productivity shocks $\{z_{i,t}\}_{i\in\mathcal{I}_s,\forall s\in\mathcal{S},t\in[0,\infty)}$, households' actions solve the households' utility maximization problems, aggregator's actions solve its profit maximization problem. Additionally, the equilibrium distribution is time-invariant, i.e. Equation (2.4) follows

$$0 = -\partial_a\left[s(a,z,t)\,g(a,z,t)\right] - \partial_z\left[\mu(z)\,g(a,z,t)\right] + \frac{1}{2}\partial_{zz}\left[\sigma^2(z)\,g(a,z,t)\right].$$

All markets clear:

1. Final goods market clears

$$\int C_{i,t}dG(a,z,t) + \delta\int a_{i,t}dG(a,z) = \Pi_s C_{s,t}^{\nu_s}$$

2. Sectoral goods markets clear

$$Q_{s,t} = \sum_{i\in\mathcal{E}_s} q_{i,t} = C_{s,t} + \sum_{h\in\mathcal{S}}\left[\int x_{i,s,t}\mathbf{1}_{\{i\in\mathcal{E}_h\}}dG_h(a,z,t)\right], \forall s\in\mathcal{S}$$

3. Capital market clears

$$\sum_{s\in\mathcal{S}}\int k_{i,s,t}\mathbf{1}_{\{i\in\mathcal{E}_s\}}dG_s(a,z) = \int a_{i,t}dG(a,z)$$

4. Labor market clears

$$\sum_{s\in\mathcal{S}}\int l_{i,s,t}\mathbf{1}_{\{i\in\mathcal{E}_s\}}dG_s(a,z) = \int \mathbf{1}_{\{i\notin\mathcal{E}\}}dG(a,z)$$

where $\mathcal{E}$ denotes the set of entrepreneurs, and $\mathcal{E}_s$ denotes the set of entrepreneurs in sector s.

In Appendix A, I characterize the aggregate saving and consumption analytically in a two-period environment. These aggregate variables not only depends on the level of financial frictions, but also their locations in the network. This tractability is lost when generalizing to the infinite horizon case, because of the presence of heterogeneous agents and financial constraints. Instead, I will use numerical examples to demonstrate the main mechanisms in the model in Section 2.1 and Section 2.2.

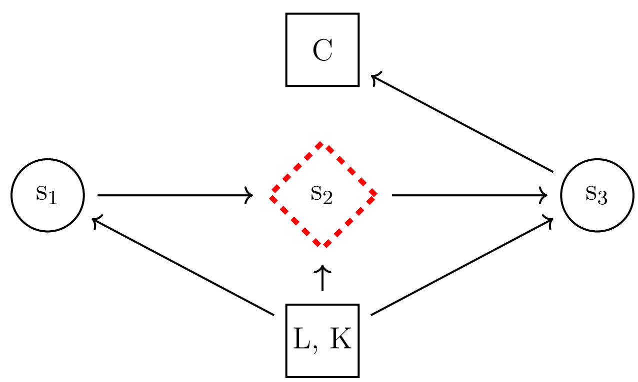
2.1 Financial Friction and Sectoral Shocks

It is well-known in the production network literature that network linkages can propagate idiosyncratic shock from one sector to another (e.g. Acemoglu et al., 2012). In this section, I show numerically how financial frictions within the network can affect the propagation of shocks and subsequent recovery. In addition to within network allocation, this exercise also generalizes the labor supply channel in Bigio and La'O (2020) by considering intertemporal capital accumulation.

To illustrate the key mechanism, consider the following vertical production network in Figure 1. There are three sectors in the economy. Sector 1 is the most upstream sector, which only uses capital and labor to produce. Sector 2 and sector 3 both also use intermediate goods, which consist of goods only from their direct upstream sectors. Sector 3 good is then used for final consumption.

In this numerical example, I solve for the transition dynamics for two levels of financial constraints in sector 2, referred as "tight" and "loose"[1]. This illustrates how the friction in a middle node of the network affects the transmission of sectoral shock to downstream sectors and eventually the aggregates.

Figure 1: Vertical Economy



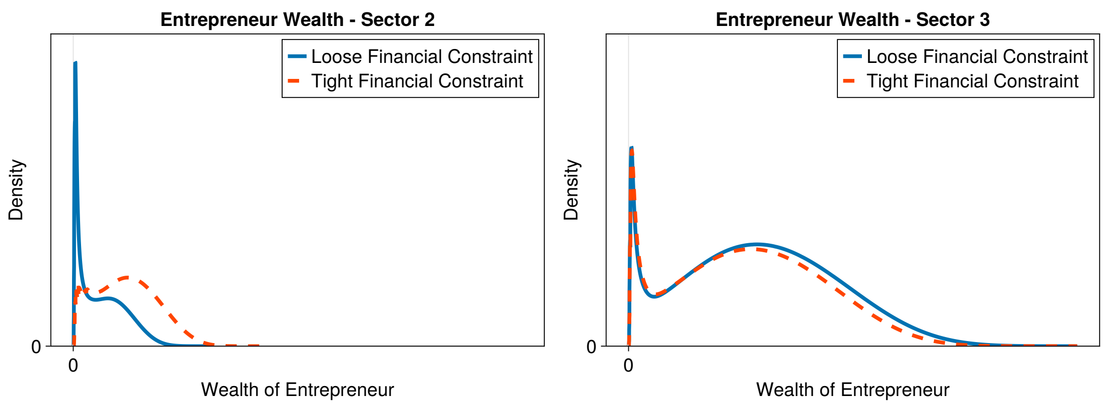
This figure shows the production network structure used in numerical example. Sector 2, marked by the red diamond shape, is the sector of interest. I solve for the transition dynamics for two levels of financial constraints in sector 2, labelled as "tight" and "loose".

Figure 2 shows the equilibrium distributions of entrepreneurs among sector 2 and 3. Financial constraints distort the selection into entrepreneurship in sector 2. Productive but poor entrepreneurs cannot operate at profitable scales, leaving rooms for unproductive but wealthy entrepreneurs. Thus the distribution in the left panel of Figure 2 shifts to the right. This impacts sector 3, its downstream, via two channels. First, higher wealth in sector 2 increases aggregate capital supply in the economy, leading to a lower equilibrium interest rate. Hence households from other sectors

[1] In the "tight" scenario, I set $\theta_2 = 1$ as in Equation 2.2. In the "loose" scenario, I set $\theta_2 = 7$. In comparison, $\theta_1 = \theta_3 = 5$, for the other two sectors. Later in Section 3.2, the median of the calibrated levels of financial constraints is 6.

save less. Notice this is not related to the network structure, but rather the levels of financial constraints in the economy as a whole. Second, lower sectoral productivity from sector 2 translates into higher input cost for its downstream, limiting the scales of productions in sector 3. When the optimal scale decreases, financial constraint matters less. As a result, the equilibrium distribution in sector 3 tilts towards more productive, but relatively poor entrepreneurs.

Figure 2: Wealth Distribution of Entrepreneurs Pre-Shock



This figure shows the equilibrium distributions of entrepreneurs in sector 2 and 3 for a numerical example with the network structure described in Figure 1. Blue curves correspond to a scenario where the borrowing constraint in sector 2 is relatively loose. Red curves correspond to a scenario where the the borrowing constraint in sector 2 is relatively tight. X-axis is the wealth level of entrepreneurs. Y-axis is distribution density.

The first row of Figure 3 plots the aggregate responses to an upstream productivity shock. These responses are normalized by their steady-state values to show their relative scales. Aggregate consumption experiences a sharper decline comparing to the case where financial constraint in sector 2 is slacker. On the other hand, aggregate capital depletes less.

The middle row of Figure 3 shows the sectoral firm dynamics following shocks. Entrepreneurs in sector 2 decide to close their businesses in response to an upstream negative shock, as the price of their intermediate input rises, hurting their profitability. This response is more pronounced under a tighter financial constraint, which also makes the transition back to steady-state slower in terms of the number of firms. This finding is consistent with Buera and Shin (2013)'s concept of "the persistence of history," although the shock studied here is sectoral rather than aggregate. Due to these firm closures, the price of the sector 2 good increases, which in turn hurts the profitability of entrepreneurs in sector 3, leading to a cascade of closures. I refer to this as the "transmission effect."

On the other hand, sector 3 experiences sharper declines and a slower recovery when the financial constraints in its upstream sector are looser. This counterintuitive response can be attributed to the asymmetric behavior of exits and the extent of financial constraints, as shown in the bottom row of Figure 3. Facing a negative shock, more entrepreneurs in sector 2 become financially constrained in an environment with a relatively loose constraint, as they hold less wealth in equilibrium. However, most of these entrepreneurs do not choose to close their businesses, as shown in the left column of the middle row. This is because, under looser constraints, entrepreneurs with limited wealth are able to operate at higher scales. Following a negative upstream shock, despite being financially constrained, they can still maintain their businesses at sizes that generate enough profits to outweigh their outside option, which is earning a worker's wage. In contrast, in a tight financial constraint environment, even with more wealth, entrepreneurs are more prone to being constrained, as they need to maintain larger scales to compensate for their lower productivities.

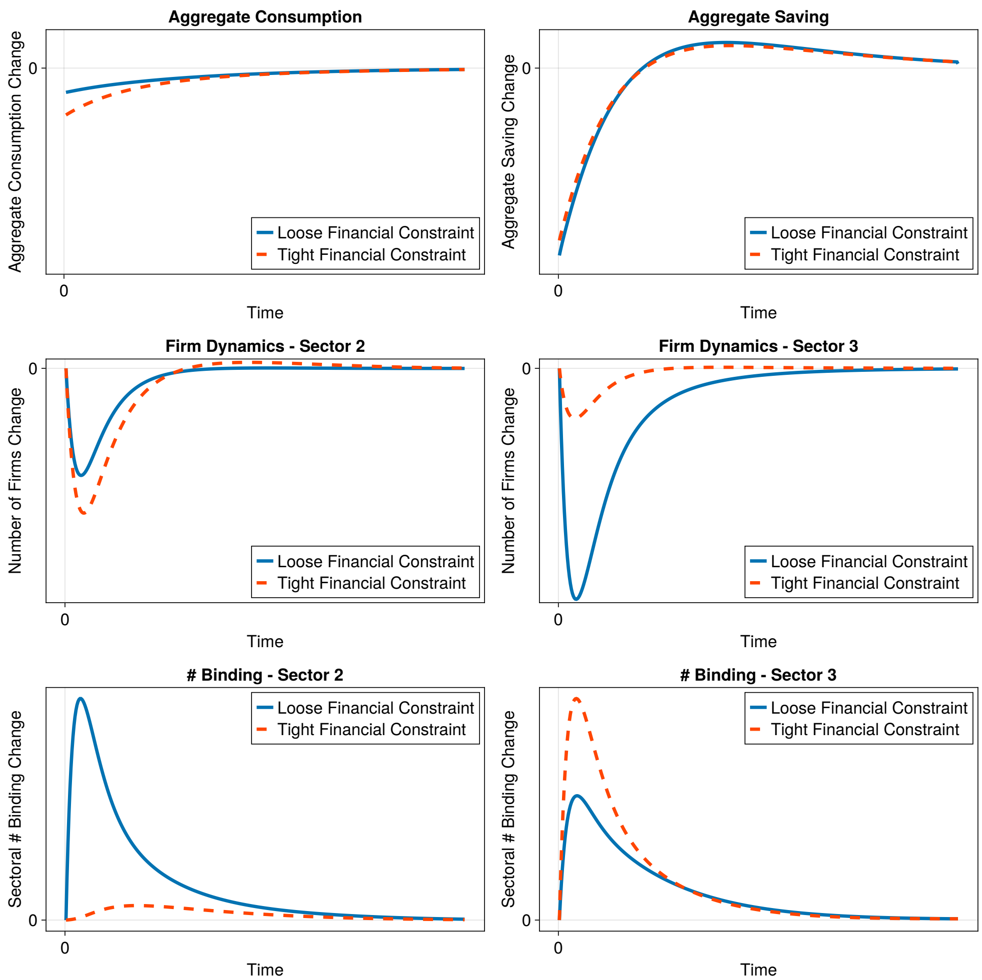
The same argument applies to the dynamics in sector 3 but in the opposite direction. This results from the fact that the size distributions in Figure 2 show opposite patterns for sectors 2 and 3 under the two environments. Intuitively, with tighter financial constraints in its upstream sector, the downstream sector behaves as if it's financial constraint is relaxed due to the equilibrium forces described above. This means more entrepreneurs are on the right tail of the productivity distribution, leading to smaller responses. I refer to this as the "cleansing effect," as it operates through changes in the extensive margin, depending on the equilibrium distribution.

After a negative shock, two groups of entrepreneurs endogenously close their businesses: unproductive entrepreneurs due to lower profits and poorer entrepreneurs due to financial constraints. Changes in the aggregate environment simultaneously affect these two margins. The intensities of these effects depend on the pre-shock distribution and the persistence of the shocks. Higher intensity in the former raises sectoral productivity, and vice versa.

In the example provided in Figure 3, the "transmission effect" and the "cleansing effect" work against each other. The "transmission effect" operates through network linkages, while the "cleansing effect" works through changes in the distribution. To separate these two forces, I examine negative shocks with varying persistence. Intuitively, a persistent shock has a larger impact through distributional changes as entrepreneurs' wealth depletes more. Therefore, with a low-persistence shock, I expect the "transmission effect" to dominate, as there is minimal impact on the distribution. Conversely, with a high-persistence shock, the "cleansing effect" should play a more significant role.

This is indeed the case as shown in Figure 4. The left column shows the firm dynamics in sectors 2 and 3 in response to a negative upstream productivity shock with low persistence. Both sectors exhibit larger negative responses when the financial constraint in sector 2 is tight, consistent with the "transmission effect." The right column shows the same dynamics in response to a high-persistence shock. As the shock endures, entrepreneurs in both sectors experience greater wealth depletion due to consumption smoothing. Depending on the equilibrium distribution, the "cleansing effect" leads to opposite comparative statics in sectors 2 and 3.

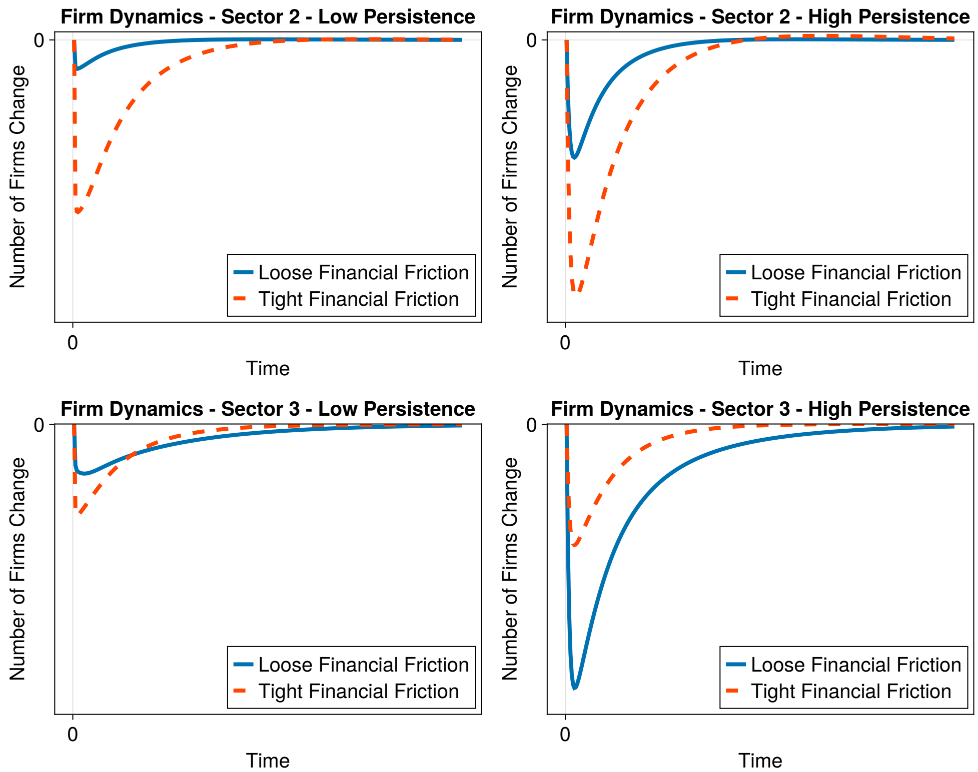
Figure 3: Time Paths Following Sector 1 Productivity Shock



This figure shows the transition dynamics following a negative productivity shock to sector 1 for a numerical example with the network structure described in Figure 1. Blue curves correspond to a scenario where the borrowing constraint in sector 2 is relatively loose. Red curves correspond to a scenario where the the borrowing constraint in sector 2 is relatively tight. The top row shows the transition dynamics of aggregate consumption (left) and aggregate saving (right). The middle row shows the transition dynamics of the number of firms in sector 2 (left) and sector 3 (right). The bottom row shows the transition dynamics of the number of financially binding firms in sector 2 (left) and sector 3 (right). All values are normalized by their steady state levels. X-axis is time. Y-axis is level. Y-axes on the same row are normalized to the same scale.

In the model, two opposing forces influence the impact of financial constraints on firm dynamics. On one hand, tight financial constraints prevent poorer households from becoming entrepreneurs. On the other, they enable wealthier but less productive households to start their own businesses. Consequently, while the effect of financial constraints on aggregate consumption is negative due to their distortion of aggregate productivity, the impact on firm entry and exit remains ambiguous and depends on the distribution of households' wealth.

Figure 4: Firm Dynamics and Persistence of Shocks - Financial Friction



This figure shows the transition dynamics following a negative productivity shock to sector 1 for a numerical example with a network structure described in Figure 1. Blue curves correspond to a scenario where the borrowing constraint in sector 2 is relatively loose. Red curves correspond to a scenario where the the borrowing constraint in sector 2 is relatively tight. The top row shows the transition dynamics of the number of firms in sector 2 with a low persistence shock (left) and a high persistence shock (right). The top row shows the transition dynamics of the number of firms in sector 3 with a low persistence shock (left) and a high persistence shock (right). All values are normalized by their steady state levels. X-axis is time. Y-axis is level. Y-axes on the same row are normalized to the same scale.

2.2 Production Network and Aggregate Shocks

In this section, I use a numerical example to demonstrate that the structure of the production network can also lead to differing transition dynamics for aggregate and sectoral outcomes following aggregate shocks.

To isolate the impact of the network structure, I study transition dynamics in two distinct networks with identical setups in other dimensions. Figure 5 shows the network structure of interest. Sector 1, being the upstream sector, supplies intermediate goods to sector 2 and 3, but does not contribute to the final consumption. Sector 3 uses output from sector 2 as input and has a greater weight in the final consumption bundle. The red dashed arrow represents this relationship, which I refer to as the “connected economy.” To highlight the effect of the network, I also examine an alternative setup where the linkage between sectors 2 and 3 is completely broken, making sector 2 and 3 disconnected. I refer to this configuration as the "disconnected economy." Naturally, disconnecting sector 2 from sector 3 halts the transmission of sectoral shocks through that channel. However, the purpose of this numerical exercise is to determine whether this change in the network structure affects the transition dynamics after an aggregate productivity shock impacting all sectors. To isolate the network structure effect, I assume sector 1 is not subject to financial frictions.

Figure 5: Connected/Disconnected Economy

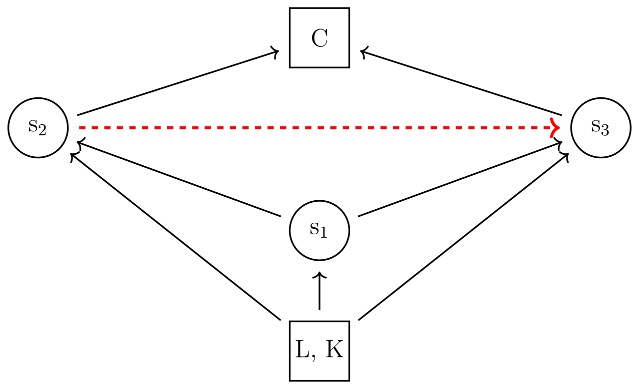
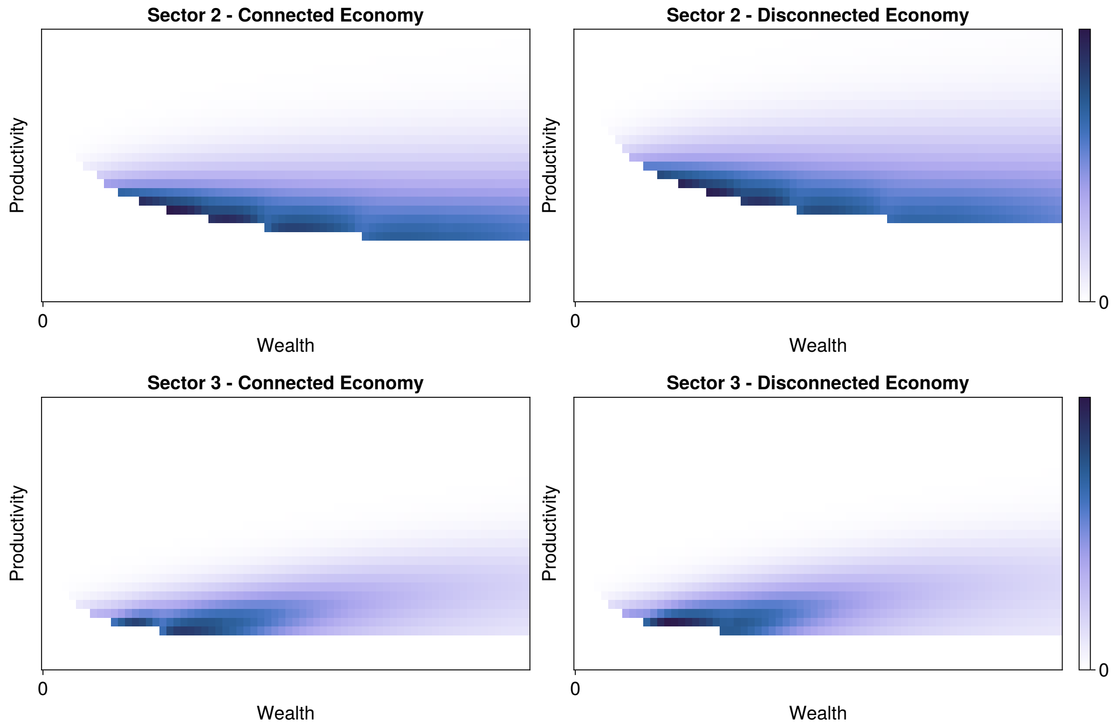


Figure 6 shows the equilibrium distributions of entrepreneurs in sectors 2 and 3 for the "connected economy" and the "disconnected economy." Comparing the left and right figures in the first row, we see that when the linkage is broken, the distribution of entrepreneurs in sector 2 shifts upward toward regions with higher productivity. This means that, with the same level of wealth, households need to be more productive to run a firm that is profitable enough. Similarly, with the same productivity level, households need more wealth to operate at a scale that surpasses their outside option of earning a worker's wage. This result stems from the decreased demand when sector 3 stops using sector 2's output in its production. Lower demand leads to lower prices, which reduce profit margins and enforce a stricter selection into entrepreneurship.

Turning to sector 3 in the "disconnected economy," more entrepreneurs are concentrated in the lower-left corner of the distribution, as shown in the bottom-right panel of Figure 6. This indicates that, at equilibrium, entrepreneurs in sector 3 are closer to the boundary of financial constraints. This outcome results from the stricter selection into entrepreneurship in sector 2. Higher productivity in sector 2 exerts downward pressure on the price of sector 3 goods, thereby suppressing its production scale.

Figure 6: Joint Distribution of Entrepreneurs Pre-shock

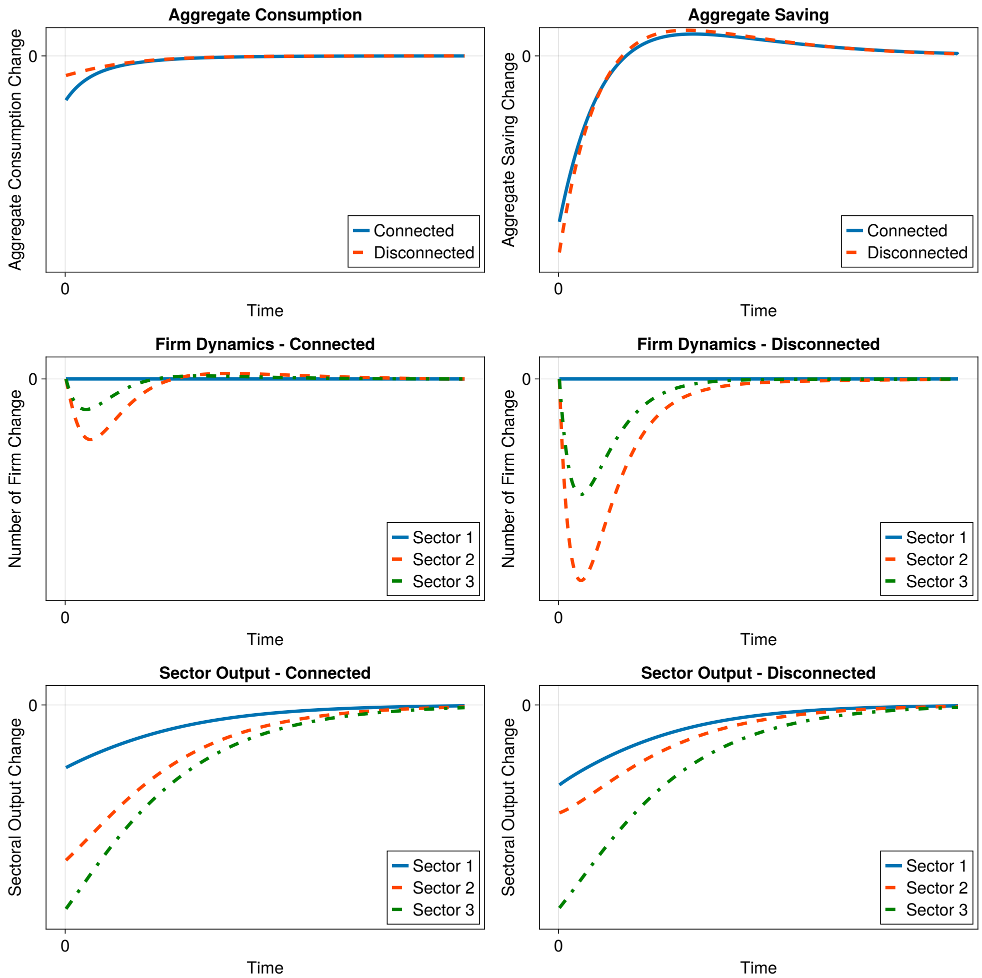


This figure shows the equilibrium distributions of entrepreneurs in sector 2 and 3 for a numerical example with a network structure described in Figure 5. The left column shows the joint distribution of productivity and wealth in the connected economy for entrepreneurs in sector 2 (top) and sector 3 (bottom). The right column shows the joint distribution of productivity and wealth in the disconnected economy for entrepreneurs in sector 2 (top) and sector 3 (bottom)t. X-axis is the wealth level of entrepreneurs. Y-axis is productivity of entrepreneurs. Brighter colors denote areas with more mass. For the same row, the gradient of color is normalized to be the same.

The top row of Figure 7 shows the transition paths of aggregate consumption and savings following a negative shock to aggregate productivity. The "connected economy" experiences a larger decline in aggregate consumption but smaller capital depletion compared to the "disconnected economy." The production network not only propagates sectoral shocks, as shown in the previous section, but also amplifies the consumption response to aggregate shocks.

The middle row compares the firm dynamics in all three sectors between the two economies. The series are normalized by the numbers of firms at steady-state. First, because of the absence of financial constraints, the firm dynamics in sector 1 is negligible. It is apparent that both sectors experience more exits in the disconnected economy. This is due to the differences in equilibrium distributions, as shown in Figure 6, i.e. the "cleansing effect". In both sector 2 and 3, entrepreneurs concentrate in regions closer to the borrowing constraints, making them more prone to closures after wealth depletion.

Figure 7: Time Path Following Aggregate Productivity Shock

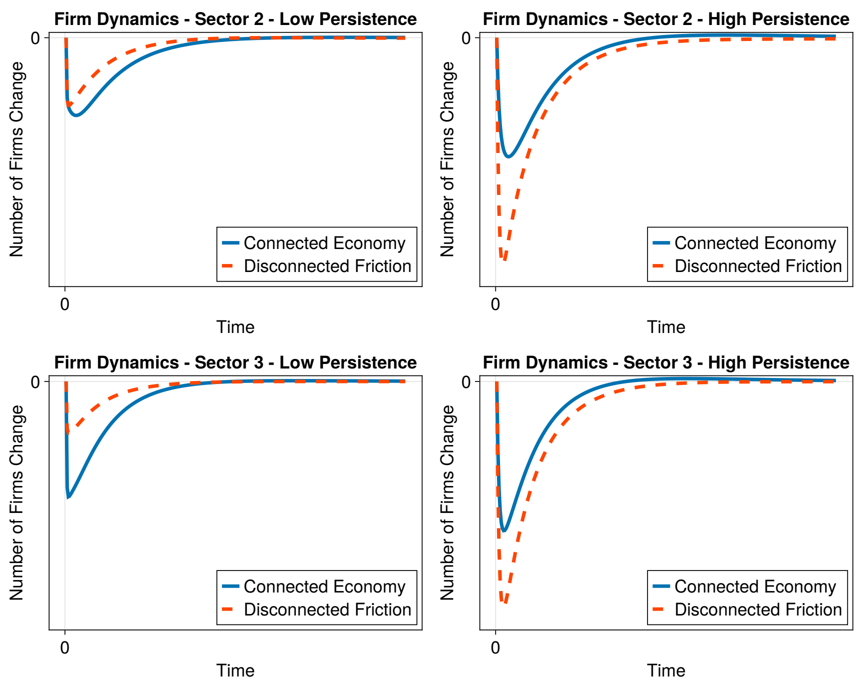


This figure shows the transition dynamics following a negative aggregate productivity shock for a numerical example with a network structure described in Figure 5. In the first row, blue curves correspond to the "connected economy", red curves correspond to the "disconnected economy". It shows the transition dynamics of aggregate consumption (left) and aggregate saving (right). The middle row shows the transition dynamics of the number of firms in each sector in the "connected economy" (left) and the "disconnect economy" (right). The bottom row shows the transition dynamics of output from each sector in the "connected economy" (left) and the "disconnect economy" (right). All values are normalized by their steady state levels. X-axis is time. Y-axis is level. Y-axes on the same row are normalized to the same scale.

The bottom row shows the declines in sectoral outputs, relative to pre-shock GDP. Sector 1 and 3 shows comparable drops in sizes. Sector 2 experience more decline in output in the connected economy. This showcases the network effect through intermediate demands, i.e. the "transmission effect". If we ignore the general equilibrium feedbacks, from the point of view of the midstream sector, an aggregate productivity shock is equivalent to a combination of sectoral productivity shocks on both its upstream and downstream and itself. The shock on downstream sector is propagated through the production network via decreased demand.

Like in the previous section, the "transmission effect" and the "cleansing effect" can be disentangled by studying shocks of different levels of persistence. Figure 8 shows firm dynamics in both downstream sectors in response to shocks with low and high persistence. Consistent with intuition, more exits occur when the negative shock is more persistent. For both sectors, the "connected economy" experiences more exits compared to the "disconnected economy" when the shock has low persistence. This relationship reverses with a persistent shock. Under a transitory shock, sectoral linkages introduce the "transmission effect," leading to more closures. In contrast, a persistent shock leads to greater capital depletion, where the "cleansing effect" dominates, resulting in more closures in the "disconnected economy" due to differences in equilibrium distributions, as shown in Figure 6.

Figure 8: Firm Dynamics and Persistence of Shocks - Network Structure



This figure shows the transition dynamics following a negative aggregate productivity shock for a numerical example with a network structure described in Figure 5. Blue curves correspond to the "connected economy". Red curves correspond to the "disconnected economy". The top row shows the transition dynamics of the number of firms in sector 2 with a low persistence shock (left) and a high persistence shock (right). The top row shows the transition dynamics of the number of firms in sector 3 with a low persistence shock (left) and a high persistence shock (right). All values are normalized by their steady state levels. X-axis is time. Y-axis is level. Y-axes on the same row are normalized to the same scale.

This exercise, along with the one in the previous section, highlights the complex implications of production networks on transition dynamics. The presence of production networks introduces the "transmission effect," which propagates the transition dynamics from upstream to downstream. On the other hand, the networks and financial constraints jointly determine the equilibrium distribution of entrepreneurs, affecting the intensity and direction of the "cleansing effect."

2.3 Discussion of Limitations

Despite of the rich mechanisms captured in this model, there are several compromises in the assumptions. These limitations affect the interpretations of the model and subsequent quantitative results.

Frictionless Occupational Choice Both occupational choice and production are intratemporal. This is a common setup in models featuring occupational choice (Buera, Kaboski, and Shin, 2011; Buera and Shin, 2013, etc) and production network models (Acemoglu et al., 2012; Liu, 2019; Bigio and La'O, 2020, etc). Households can change their occupations frictionlessly each instance. This greatly simplifies the problem as the households' intertemporal decisions only involve the canonical consumption and saving problems.

Comparing to the firm entry and exit models (Hopenhayn, 1992; Vereshchagina and Hopenhayn, 2009; Hopenhayn, 2014), where entrepreneurs incur fixed costs when either entering or exiting, my model omits the frictions on the front of business creation and destruction. The effect of this omission is ambiguous. On the one hand, costly entry/exit hinders the on-shock reactions of the entrepreneurs, leading to a wait-and-see approach. This could decrease the number of exits. On the other hand, the cost also discourages new entry, as the expected cash-flow must offset the upfront cost. This could decrease the number of entries.

This omission has important implications on welfare interpretation and policy analysis. The effect of financial frictions on the aggregates is purely due to its negative impact on productivity and subsequent general equilibrium feedbacks. Thus, firm entries and exits do not have clear welfare interpretation as no resources are wasted during this process. On the other hand, typical firm subsidy programs, e.g. credit intervention, often take into consideration the cost associated with firm closures. In my model, this force is partially reflected in entrepreneurs' wealth depletion via consumption smoothing.

The Lack of Firm Borrowing Intratemporal production decisions rule out the possibility of borrowing by firms. Indeed, because capital rental has to be paid off within the same instance, there is no inter-period debt. Thus this model is unsuited to study the endogenous take up of loans provided by fiscal programs. In later quantitative exercises, I treat the actual loan take up as exogenous and explore alternative designs. I will discuss its implication on policy interpretation in detail when I describe how I model the shocks in Section 4.3.

Frictionless Labor Market In the model, labor market is frictionless. This is in contrast to the frictional labor search model (Mortensen and Pissarides, 1994), where the match and rematch process between workers and firms are subject to costly search and negotiation process. In another strand of literature, the pair between firms and employees has value (Davis and Haltiwanger, 1992; Jacobson, LaLonde, and Sullivan, 1993; Fujita and Moscarini, 2017). The closing and reopening of firms thus are value destroying. The inclusion of labor market frictions can prolong the transition dynamics of the model and create additional welfare loss due to firm closures. These extensions are relevant to policy analysis and open to future research.

Long-term Consequences This model does not feature growth. Due to this omission, my model is more suited to study the impact of firm subsidy programs during crises. Taking the CARES Act during COVID-19 as an example, the primary goal is to support failing businesses to prevent further economy-wide cascades. Various inefficiencies, such as moral hazard problems and distortions on long-term productivity growth, are of second order during this period. This further separates this paper from existing studies on subsidy programs, for example, Liu (2019) on industrial policies, and Li and Li (2020) on credit support programs.

3 Quantitative Analysis

As shown in the previous sections, the quantitative performance of this model depends on the structure of the production network, sectoral production functions, and sector-specific financial frictions. In this section, I describe the calibration process of the model in detail.

I assume the households follow the same CRRA utility function:

$$U(C) = \frac{C^{1-\sigma_u}}{1-\sigma_u}.$$

The parameters in the baseline model can be divided into the following groups: Household preference: $\{\sigma_u, \rho\}$, production technology $\{\alpha_s, \beta_s, \gamma_s\}_{s\in\mathcal{S}}$, δ, exogenous stochastic process $\{\mu, \sigma\}$, and the level of financial friction for each sector $\{\theta_s\}_{s\in\mathcal{S}}$. Due to data limitation, I assign the preference parameters, and level of capital depreciation δ, according to prevailing values in the macroeconomics literature. Then I estimate the production function of each sector using a combination of Compustat firms and BEA aggregate reports. Because the production function estimation is independent from the structure of the model, I refer to it as the "external calibration". For productivity process, $z_{i,t}$, and financial friction parameters $\{\theta_s\}_{s\in\mathcal{S}}$, I calibrate them using Simulated Method of Moments (SMM). I refer to it as the "internal calibration".

In the baseline model, I set $\sigma_u = 2$ and $\rho = 0.08$ to match the common values in the literature (e.g. Buera, Kaboski, and Shin, 2011; Moll, 2014).

3.1 External Calibration

The production function estimation is done in two steps. First, I follow Gandhi, Navarro, and Rivers (2020) and estimate the production non-parametrically. The main idea is to explore the first order condition associated with the intermediate inputs. Given the gross production function

$$Y_{i,t} = F\left(k_{i,t}, l_{i,t}, m_{i,t}\right) e^{\log(z_{i,t})}$$

The first order condition with respect to intermediate goods is

$$P_t \frac{\partial}{\partial M_{i,t}} F\left(k_{i,t}, l_{i,t}, m_{i,t}\right) e^{\omega_{i,t}} \mathcal{E} = \rho_t$$

where $\omega_{i,t}$ is the persistent part of stochastic process $z_{i,t}$, and $\mathcal{E}$ is the expectation of its innovation. Taking logs and rewriting this equation,

$$\begin{aligned} s_{i,t} = \log\left(\frac{\mathbf{P} M_{i,t}}{P_t Y_{i,t}}\right) &= \log\left(D\left(k_{i,t}, l_{i,t}, m_{i,t}\right)\right) + \log\mathcal{E} - \varepsilon_{i,t} \\ \text{where } D\left(k_{i,t}, l_{i,t}, m_{i,t}\right) &\equiv \frac{\partial f\left(k_{i,t}, l_{i,t}, m_{i,t}\right)}{\partial m_{i,t}}. \end{aligned} \tag{3.1}$$

Equation (3.1) can be estimated using non-parametric regressions on the share of intermediary expenditure over sales. Once $D\left(k_{i,t}, l_{i,t}, m_{i,t}\right)$ is available, I can integrate $D(\cdot)$ to recover the production function up to a constant $\mathcal{C}\left(k_{i,t}, l_{i,t}\right)$. Then by assuming a autoregressive process of the persistent component $\omega_{i,t} = h\left(\omega_{i,t-1}\right) + \eta_{i,t}$, $\mathcal{C}\left(k_{i,t}, l_{i,t}\right)$ can be identified with the standard dynamic panel argument in Olley and Pakes (1996).

One substantial difference between the method adopted here, and other popular methods, for example Olley and Pakes (1996) and Ackerberg, Caves, and Frazer (2015), is that the production function considered here is the gross production function, whereas the methods developed around the idea of Olley and Pakes (1996) consider value-added production function. Only under strong structural assumptions, for example, intermediate goods enter the production function in a Leontief way, one can recover the gross production function from value-added production function and vice-versa.

The procedure described above returns production elasticity with respect to capital, labor and intermediate inputs. Under the functional form assumption in Section 2, these elasticities equal to the parameters on factor inputs. I estimate the production function using US firm-level data from

Compustat, the sample runs from 2000 to 2019. The logic behind this sample selection is to form a representative period of US production economy without including the period of interest, COVID-19. The estimation is done for each firm in the sample. I then take the median of the estimates by sector. The estimation results are presented in Table 1. There are rich heterogeneity in terms of capital, labor and intermediate good intensities across sectors.

Table 1: Production Function Estimation Results

Sector ID	**Sector**	α	β	γ	**Profit Share**
1	Natural Resources	0.30	0.32	0.35	0.34
2	Construction	0.14	0.20	0.63	0.19
3	Durable Manufacturing	0.17	0.28	0.50	0.16
4	Nondurable Manufacturing	0.18	0.27	0.50	0.25
5	Trade	0.15	0.21	0.62	0.24
6	Information	0.17	0.32	0.43	0.33
7	Real Estate	0.24	0.31	0.40	0.39
8	Professional	0.13	0.28	0.50	0.15
9	Education	0.18	0.28	0.44	0.11
10	Food and Entertainment	0.24	0.26	0.42	0.17

This table shows the results from the production function estimation. α corresponds to capital input, β corresponds to labor input, γ corresponds to intermediate inputs. The estimation uses firm-level panel data from Compustat from 2000 to 2019. Profit share is the ratio between gross operating surplus and total industry output from BEA.

A limitation of this approach is the sample selection bias, with public firms disproportionately representing larger firms. Under the market structure specified in the model, the reason why they have larger sizes in equilibrium can be attributed to the fact that the returns-to-scale of their production technologies are closer to 1, as suggested by the results in Table 1.

To account for this sample bias, in the second step, I use the cost decomposition data from BEA's input-output account. I consider the following production function structure:

$$y_{i,s,t} = z_{i,s,t} F_s \left(l_{i,s}, k_{i,s} x_{i,s} \right) = z_{i,s,t} \left(k_{i,s}^{\alpha_s} l_{i,s}^{\beta_s} x_{i,s}^{\gamma_s} \right)^{\eta_s}$$

where the factor weights α_s, β_s and γ_s are given by the non-parametric estimation above. Parameter η_s governs the returns-to-scale levels. The assumption with this setup is that the estimation outlined above captures the sectoral production function up to the relative weights of input factors. To gain a plausible estimation of the sectoral levels of returns-to-scale, regardless of their status of being public traded or not, I choose η_s to match the profit margins reported by BEA, which are listed in the last column of Table 1. This approach follows the literature studying quantitative models with occupational choice (Buera, Kaboski, and Shin, 2011) and production network (Bigio and La'O, 2020), where they use BEA data to back out returns-to-scale parameters.

3.2 Internal Calibration

I estimate the productivity process and levels of financial frictions simultaneously in the spirit of Catherine et al. (2022). Given a set of parameters Θ, which includes the tightness of borrowing constraints by sector, $\{\theta_s\}_{s\in S}$, and $\{\nu, \sigma\}$, I simulate a panel of N firms for T years starting from the steady-state. The estimated parameters minimize the distance of simulated moments, $\hat{\mathbf{m}}(\Theta)$, and data moments, $\mathbf{m}$.

$$\hat{\Theta} = \min_{\Theta} \left(\mathbf{m} - \hat{\mathbf{m}}(\Theta)\right)' \mathbf{W} \left(\mathbf{m} - \hat{\mathbf{m}}(\Theta)\right) \tag{3.2}$$

The weighting matrix $\mathbf{W}$ is the inverse of the variance-covariance matrix of the data moments, which is computed via bootstrapping with replacement on all firms.

The data moments associated with the levels of financial constraint come from the empirical evidence of Chaney, Sraer, and Thesmar (2012). They estimate the reduced-form relationship between firm capital investment and real estate values. The idea is that changes in real estate values will affect firm behavior through the channel of financial constraint, as real estate assets are often pledged as collaterals in corporate borrowing. The their main specification is

$$\frac{i_{i,t}}{k_{i,t-1}} = b_0 + b_1 \cdot \frac{REValue_{i,t}}{k_{i,t-1}} + \text{controls}_{i,t} + a_i + c_t + v_{i,t}. \tag{3.3}$$

where $REValue_{i,t}$ is real estate value of firm i at time t. a_i and c_t denote firm and time fixed effects respectively. b_1 is the coefficients that reflects the level of financial constraint. I estimate (3.3) by sector using firm-level data from Compustat from 1993 to 2019 following their data construction.

To run the same regression with simulated data, I need to first map the variables in regression (3.3) to the variables in the model. Comparing Equation (3.3) to Equation (2.1), my model is a simplification of firm investment models used in the corporate finance literature e.g. Hennessy and Whited (2005). In my model, capital accumulation is done on the household side in the form of wealth accumulation. To find a similar concept as capital expenditure used in regression (3.3), I proxy $\frac{i_{i,t}}{k_{i,t-1}}$ with $\frac{k_{i,t}-k_{i,t-1}}{k_{i,t-1}}$, which is the growth of capital input. The idea is to mimic the definition of investment in a model with actual capital accumulation, $k_{i,t} = (1-\delta)\, k_{i,t-1} + i_{i,t}$.

For variations in the real estate values, I use entrepreneurs' wealth. In particular, I assume for firm i in sector s, it faces the following borrowing constraint when making intratemporal operation decisions:

$$k_{i,t} \leq \theta_s \left(a_{i,t}\epsilon_{i,t}\right), i \in \mathcal{I}_s,\ \forall s \in \mathcal{S}, \tag{3.4}$$

where $\epsilon_{i,s,t}$ is a firm-specific, i.i.d unexpected valuation shock that only matters during the borrowing process. A good draw of ϵ means the firm can borrow more, and vice-versa. The assumption behind this specification is that real estate assets and entrepreneurs' own wealth, have the same pledgeability when used as collaterals. In practice, firms generally operate with leverage. Real estate assets, which are not fully financed by equity, have lower pledgeability than equity itself. This contributes to the difference in the levels of financial constraint parameters from my estimates and those from Catherine et al. (2022)[2].

When entrepreneurs first start their firms, the borrowing constraint is more likely to be binding. This is because household's wealth is relatively slow moving. However, the firms included in Compustat are mostly mature firms. For this reason, when running the equivalent regression to (3.3) with simulated data, I only include firms with at least 5 years of age[3]. This requirement makes the entrepreneurs in my simulated regression closer to the definition of firms in Chaney, Sraer, and Thesmar (2012). When an entrepreneur is lucky enough to draw consecutive periods of high productivity, she has accumulated enough wealth such that she become very unlikely to quit in the future. In other words, these entrepreneurs are far from the extensive margin of occupational choice, so that their behaviors resemble established firms.

For productivity process $z_{i,t}$, I assume $\log(z_{i,t})$ follows the same Ornstein–Uhlenbeck process for all sectors:

$$d\log(z_{i,t}) = \mu(\bar{z} - z_{i,t})\,dt + \sigma dW_{i,t}. \tag{3.5}$$

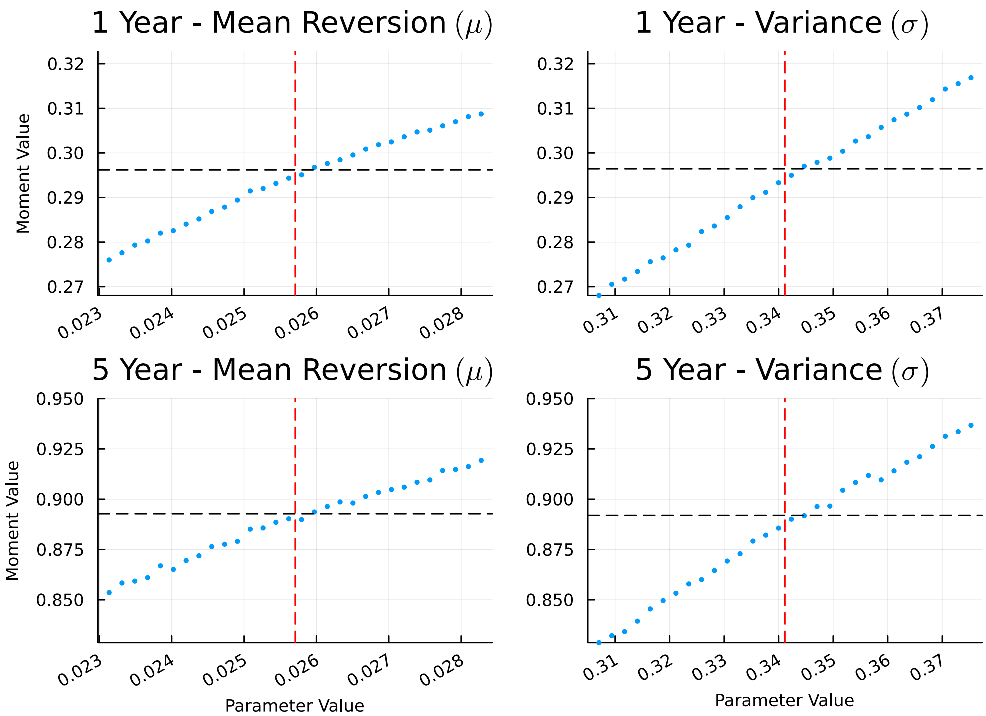
I then employ the logic from Midrigan and Xu (2014) to use the volatility of the change of sales at different horizon to estimate the parameters in (3.5). In my data sample, the standard deviations of one year change in log sales, $(\log \text{sale}_{i,t} - \log \text{sale}_{i,t-1})$ and five year changes in log sales, $(\log \text{sale}_{i,t} - \log \text{sale}_{i,t-5})$ are 29.7% and 89.1% respectively[4]. The ratio between these statistics speaks to the mean reversion parameter, μ. In the baseline model, I assume that the parameters for the productivity process are the same for all sectors. This is mainly due to data limitations. Statistics on five-year changes cannot be reliably estimated in a relatively small sector among the public firms. This means the weights associated with these statistics become small, contradicting the benefits of adding them in the first place. On the other hand, the heterogeneous effect of being selected into entrepreneurship due to different production technologies and preferences is partially captured by the sectoral distributions at equilibrium. This helps mitigate the restriction posted by a common productivity process.

[2]Their structural form of the borrowing constraint is slightly different from Equation (2.2). They found that $\{\theta_s\}$ ranges from 0.20 to 0.25: each \$1 of capital provides about \$0.20 of debt capacity.

[3]This means 5 consecutive years of operation without exiting

[4]These statistics are different from those reported by Catherine et al. (2022) as the sample period is different, and the sample winsorization is done at sectoral level.

Figure 9: Calibration - Productivity Process

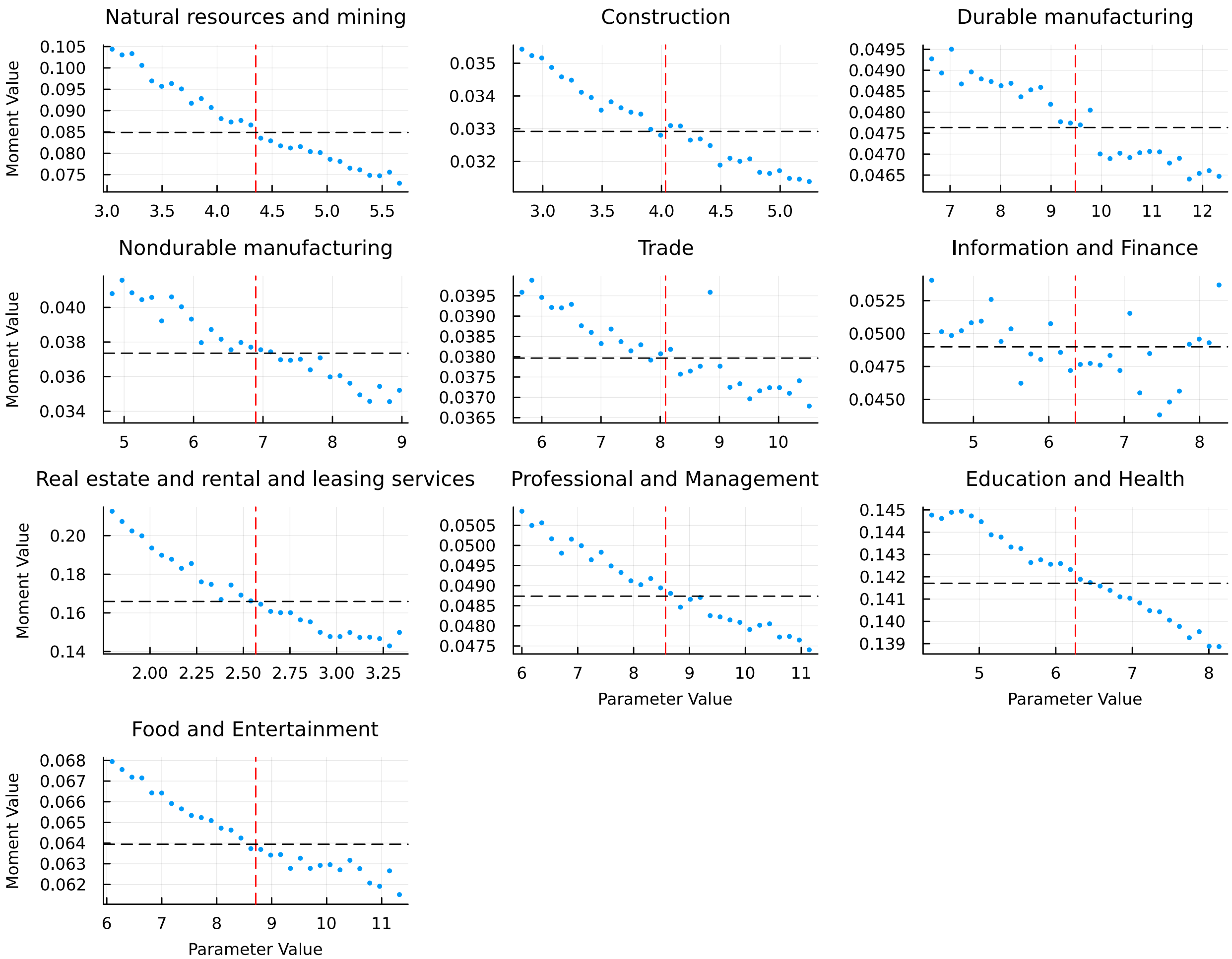


This figure shows the calibrated values and corresponding data moments, and the model's sensitivity around the calibrated values. Each subfigures shows how the sale volatilities at different horizons, specified in the subtitles, co-move with productivity process parameters around the calibrated values. The x-axis is the range of parameters values, the y-axis is the value of moments. The blue dots are simulated parameter-moment pairs. The red lines are the calibrated values of parameters. The black dashed lines are the moments in the data. The first column of subfigures for sale growth volatility at 1-year and 5-year horizons is due to variations in μ, the second column is due to variations in σ.

In Figure 9 and 10, I show the estimated parameters of interest and illustrate the sensitivity of simulated moments with respect to model parameters in the neighborhood of calibrated values. To obtain these results, I first calibrate the model as described above and obtain a set of parameters $\hat{\Theta}$. Then for $\hat{\theta} \in \hat{\Theta}$, I vary $\hat{\theta}$ around its calibrated value while keeping $\tilde{\theta} \in \hat{\Theta} \setminus \hat{\theta}$ fixed. For this new set of parameters, $\tilde{\Theta}$, I solve the model for partial equilibrium whilst keeping aggregated prices. Thus this exercise omits the general equilibrium feedbacks between sectors. For this reason, I only report the sensitivity of regression coefficients from (3.3) with respect to the levels of financial constraints in the corresponding sectors.

From Figure 9, volatilities of simulated sale growth at 1 year and 5-year horizons have monotonic relationships with the parameters in productivity process, expressed in Equation (3.5). This relationship suggests that these two parameters are locally precisely identified.

Figure 10: Calibration - Sectoral Financial Constraints



This figure shows the calibrated values and corresponding data moments, and the model's sensitivity around the calibrated values. Each subfigures shows how the data moment, coefficients b_1 in reduced-form regression (3.3), in the sector listed in the subtitle, co-move with sectoral financial constraint parameters around the calibrated values. The x-axis is the range of parameters values, the y-axis is the value of moments. The blue dots are simulated parameter-moment pairs. The variations are due to changes in the tightness of borrowing constraints in corresponding sectors. The red dashed lines are the calibrated values of parameters. The black dashed lines are the moments in the data.

From Figure 10, the sensitivities of simulated sectoral regression coefficients from Equation (3.3) show a similar picture. The coefficients have clear monotonic relationships with the parameters in all sectors except Information and Finance. The monotonic decreasing relationships between the tightnesses of borrowing constraints and the marginal effects of relaxing them are not trivial. From the specification in Equation (3.4), one may expect the regression coefficients to be larger when the borrowing constraint is looser, i.e. higher θ_s. However, with a higher θ_s, it becomes less likely for the constraint to bind. The coefficient of interest, b_1 in regression (3.3) is essentially the within-firm marginal effect of variations in entrepreneurs' wealth. When entrepreneurs' constraint rarely bind, the estimations of b_1 tilt towards zero. In other words, in a perfect credit benchmark ($\theta_s \to \infty$), the estimation of b_1 should be a precisely estimated zero. This relationship is not monotonic. In a scenario where financial constraints are extremely tight, the estimates of b_1 increase with θ_s as all entrepreneurs are constrained and the marginal effect is $\theta_s = b_1$. This non-monotonic pattern reflects the fact both production network and financial frictions shape the steady-state distribution of entrepreneurs in all sectors, echoing the discussions in Section 2.1 and Section 2.2.

The estimated levels of financial constraints show great heterogeneity across sectors. Food and Entertainment has a large estimated parameter, $\theta_{10} = 8.8$, and the comparative statics in the neighborhood show a monotonic decreasing pattern. This means the financial constraint in Food and Entertainment is relatively loose, which reflects the small coefficient obtained from running regression (3.3). On the contrary, Real estate and Rental Services has a small estimated parameter, indicating the responses estimation in regression (3.3) is large.

With this calibration, I now turn to the COVID-19 episode and study the responses of the model during crisis and the effect of corresponding fiscal policies.

4 Transition Dynamics

I consider a combination of aggregate and sectoral shocks to examine model's impulse responses during the COVID-19 episode and evaluate the performance of the PPP. Finally, I will explore improvements on the actual policy by considering alternative fund distribution between sectors.

Due to the large number of state variables in the model, solving the full transition dynamics with all the prices becomes challenging even with 10 sectors. I follow the sequential-space Jacobian method developed by Auclert et al. (2021) to solve for the transition dynamics around the steady-state.

The equilibrium conditions along the transition path, after any shock, can be characterized using the market clearing conditions in Definition 1:

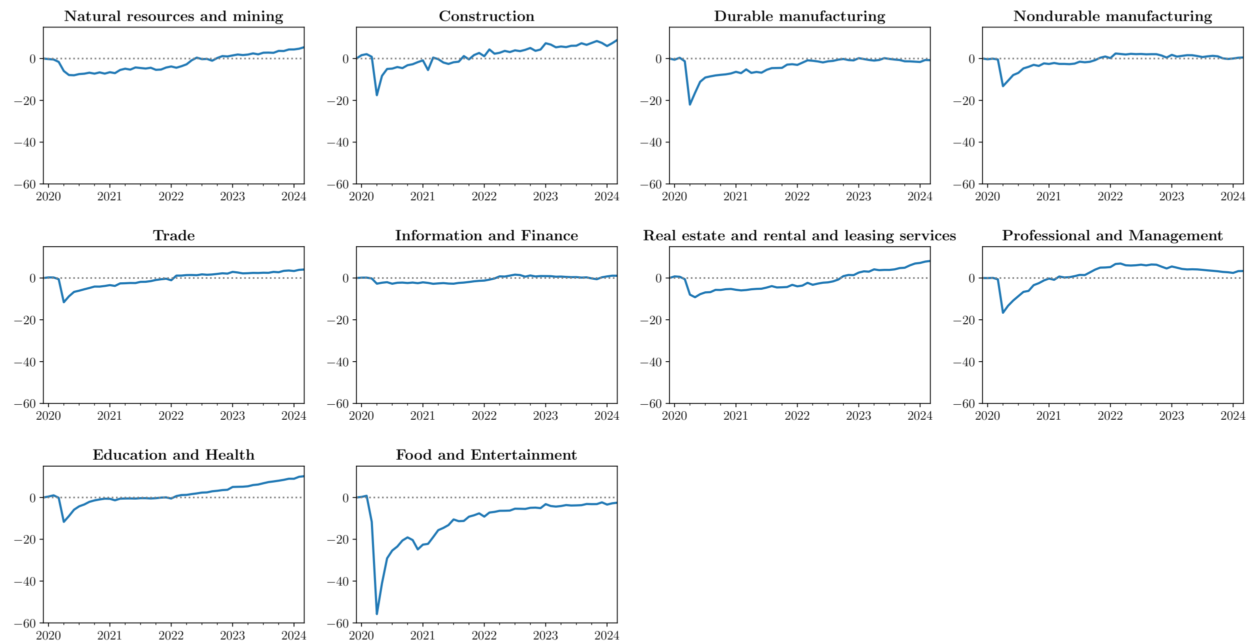
$$\mathbf{F}(\mathbf{X}, \mathbf{Z}) = \mathbf{0}$$

where $\mathbf{F}$ is a function that gathers the market clearing conditions, $\mathbf{X}$ is the set of variables that are endogenous in the model. $\mathbf{Z}$ is the set of exogenous shocks. Here I assume the number of equilibrium conditions in $\mathbf{F}(\cdot)$ is the same as the number of unknowns $\mathbf{X}$. This means the Jacobian matrix $\mathbf{F}_{\mathbf{X}}$ is locally invertible. Then the general equilibrium responses of the unknowns to any shock $\mathbf{Z}$ can be expressed using the implicit function theorem:

$$d\mathbf{X} = \mathbf{F}_{\mathbf{X}}^{-1}\mathbf{F}_{\mathbf{Z}}d\mathbf{Z}$$

For the detailed computation of the Jacobian matrices, $\mathbf{F}_{\mathbf{X}}$ and $\mathbf{F}_{\mathbf{Z}}$, I will direct the readers to Appendix B.

Figure 11: COVID-19 Shock on Working Hours by Sectors



This figure shows the sectoral changes in labor input, since COVID-19. The data are seasonally adjusted, sectoral total weekly working hours of production and nonsupervisory employees, aggregated at monthly frequency. Blue line plots the monthly percentage changes relative to the level in Dec 2019. Gray dotted horizontal lines denote the zeros. Data source: Current Employment Statistics (CES).

4.1 COVID-19 Episode

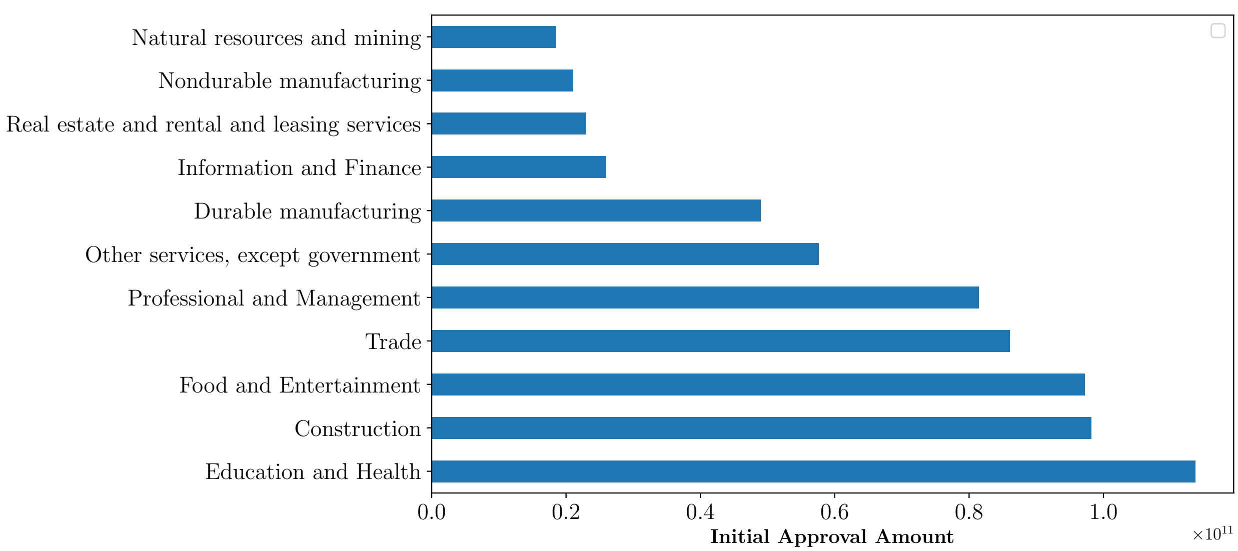
Figure 11 shows the impact of COVID-19 shocks in terms of weekly working hours in the sector. Because of social distancing and other policies that restricted physical interactions, Food and Entertainment sector experienced a sharp decline during the first months after initial shock. Other sectors, for example, Information and Finance, showed little decrease as their production activities were hardly subject to the policy restrictions. Besides the size of initial responses, the recovery from COVID-19 also differs for different sectors. In particular, Food and Entertainment sector has not yet fully recover to its level before the pandemic. On the other hand, Construction, Real estate and Education have exceeded their pre-pandemic level by 10% at mid-2024. Note that these responses are the results of a combination of shocks, including COVID-19 shocks and all other associated fiscal and monetary polices. Next, I will describe the fiscal policy I consider in this exercise and the estimation of COVID-19 shocks in the model.

4.2 Paycheck Protection Program

The subsidy program of my focus is the Paycheck Protection Program (PPP). The PPP was a $953-billion business loan program established during COVID-19. As the name suggests, the intention was to help businesses cover their payroll costs, rent, interest, and utilities. If the applicant was able to keep the employee counts, the loan could be partially or fully forgiven.

Although the PPP was not designed with sectoral concerns in mind, the final take-up by volume exhibits sectoral differences. Figure 12 shows the aggregate volume in terms of initial approval amount in loan application by sectors. The uneven distribution in volume can be driven by various factors. First of all, these sectors have different sizes pre-pandemic, and thus uneven numbers of applications. Second, because of the nature of COVID-19 and its corresponding restrictive policies, different sectors had different levels of exposure, thus resulting in different take-up volumes. However, I will treat this distribution as exogenous in my quantitative exercise as mentioned in Section 2.3. The details will be discussed in the next section where I define the shocks in the model.

Figure 12: The PPP Initial Approval Amount by Sectors



This figure plots the distribution of loan take-up volumes across sectors. Values are the initial approval amounts in the PPP application aggregated at sectoral level. Data source: U.S. Small Business Administration.

4.3 Shock Estimation

The empirical exercise begins with the construction of a series of shocks that drives the model to a similar level as observed in the data during COVID-19. Rather than matching sectoral output, I choose to use the labor moments in Figure 11. There are two reasons: first, the labor moments from Current Employment Statistics (CES) are available at monthly frequency, while the sectoral output from BEA is at quarterly frequency, and the model is estimated at monthly frequency to capture the fast-moving nature of the shock. Second, one of the most signature impact of COVID-19 on the production side was its reduction of labor supply. Thus targeting labor dynamics helps to capture the unique features of COVID-19.

In the model, COVID-19 is modeled as a combination of aggregate labor supply shock O, and preference shocks, $\{\tilde{\nu}\}$, and a cash handout program that resembles the Paycheck Protection Program, $\{\mathcal{P}\}$. Denote the stacked vector of shocks by $\mathbf{\Psi}_t$. The model experiences an unexpected shock $\{\mathbf{\Psi}_\tau\}_{\tau\in[0,T]}$ at period 0. I further assume that $\mathbf{\Psi}_t$ follows an AR(1) process with mean $\mathbf{0}$.

$$\mathbf{\Psi}_t = \boldsymbol{\rho}\mathbf{\Psi}_{t-dt} \tag{4.1}$$

The aggregate labor supply shock decreases the supply of labor of all workers by O_t[5]. It directly decreases the labor income received by workers and the available units of labor. Since labor is fully mobile across sectors, this shock affects all sectors at the same time. However, the impact of this shock varies by sector depending on the production technologies and equilibrium distributions. I use this shock to proxy for the decrease in labor supply due to lock-down and social distancing during COVID-19.

The preference shock $\{\tilde{\nu}_s\}$ alters households' preference over sectoral goods in the final consumption bundle C_t. In particular, shock $\tilde{\nu}_s$ to sector s, increases the corresponding weight in Cobb-Douglas preference function $\Pi_s C_{s,t}^{\nu_s}$. The rest of the sector weights $\{\nu_h\}_{h\in\mathcal{S}\setminus s}$ scale up or down proportionally to keep $\sum_s \nu_s = 1$ after the shock. These preference shocks represent the shift in households' consumption bundle during COVID-19 due to health and logistic concerns.

The PPP shock $\{\mathcal{P}\}$ in the model is a simplified version of the actual PPP program. In particular, because of the forgivable nature, I model the PPP shock to be a series of cash handouts to the entrepreneurs in the economy. The size of the cash handout can be different for entrepreneurs in different sectors, but are the same within one sector. Denote $\mathcal{P}_{s,t}$ as the amount of cash each household receives in sector s at time t. The fund allocated to each sector resembles the distribution in Figure 12. The program lasted for 12 months. The fund is equally divided into each month. The total size of fund maintain the same ratio as the size of the PPP to US GDP in 2019, i.e. 4%. This

[5] Since workers all supply one unit of labor inelastically at steady-state. This means with shock O_t, each worker supplies $1 - O_t$ unit of labor.

setup has the following implications.

First, the realized distribution of fund across sectors is taken as exogenous. The goal of this exercise is to evaluate the alternatives of fund distribution while keeping the overall size unchanged. Second, I ignore the government budget process. I assume the government gets an endowment of 4% GDP in 2019 and does not need to increase future taxation[6]. This means the evaluation here is too optimistic if considering Ricardian forces through households intertemporal budget constraints. However, the purpose of this exercise is to create a baseline for alternative PPP designs. By ignoring the government budget constraint, I isolate the forces induced by the network structures as well as financial constraints. Finally, the government subsidy program is treated as an unexpected shock. At period 0, along with the COVID-19 shock, all households receive the announcement that there will be cash handout at a future date. In reality, the first round of funding of the PPP was allocated between April 3 and April 16, 2020. At a monthly frequency, the anticipation effect before the announcement was made is ignored.

By varying the initial size of the shocks $\mathbf{\Psi}_0$ and the autoregressive coefficients $\boldsymbol{\rho}$, I fit the economy's impulse responses functions of labor input by sectors to the data in Figure 11. To avoid overfitting, the autoregressive parameters $\boldsymbol{\rho}$ is restricted to be positive and relatively large that all shocks eventually die out at the end of the examined time period. These restrictions rule out that scenario where one shock oscillates with big variation and dominates other shocks. I call the resulting shock vectors $\{\mathbf{\Psi}_t\}$ the *realized shocks*, and the combination of labor supply shock and preference shocks, $\{\mathbf{\Psi}_t\} \setminus \{\mathcal{P}_t\}$, the *pure COVID-19 shocks*.

The results are shown in Figure 13. The model's responses are able to fit the data both in the magnitudes and transition dynamics. In particular, with the constructed shocks, the model is able to match the initial declines in labor input at the aggregate as well as sectoral levels in Construction, Durable manufacturing, Nondurable manufacturing, Trade, and Food and Entertainment simultaneously despite the difference in scales between these sectors. In terms of the transition after the shocks, the model is able to distinctly match speedy recoveries, as in Construction, and slow recoveries, as in Food and Entertainment. Although the shocks $\{\mathbf{\Psi}_t\}$ are chosen to track the data, there are significantly more degrees of freedom than the number of parameters, $\{\mathbf{\Psi}_0, \boldsymbol{\rho}\}$[7]. This demonstrates the explanatory power of the model.

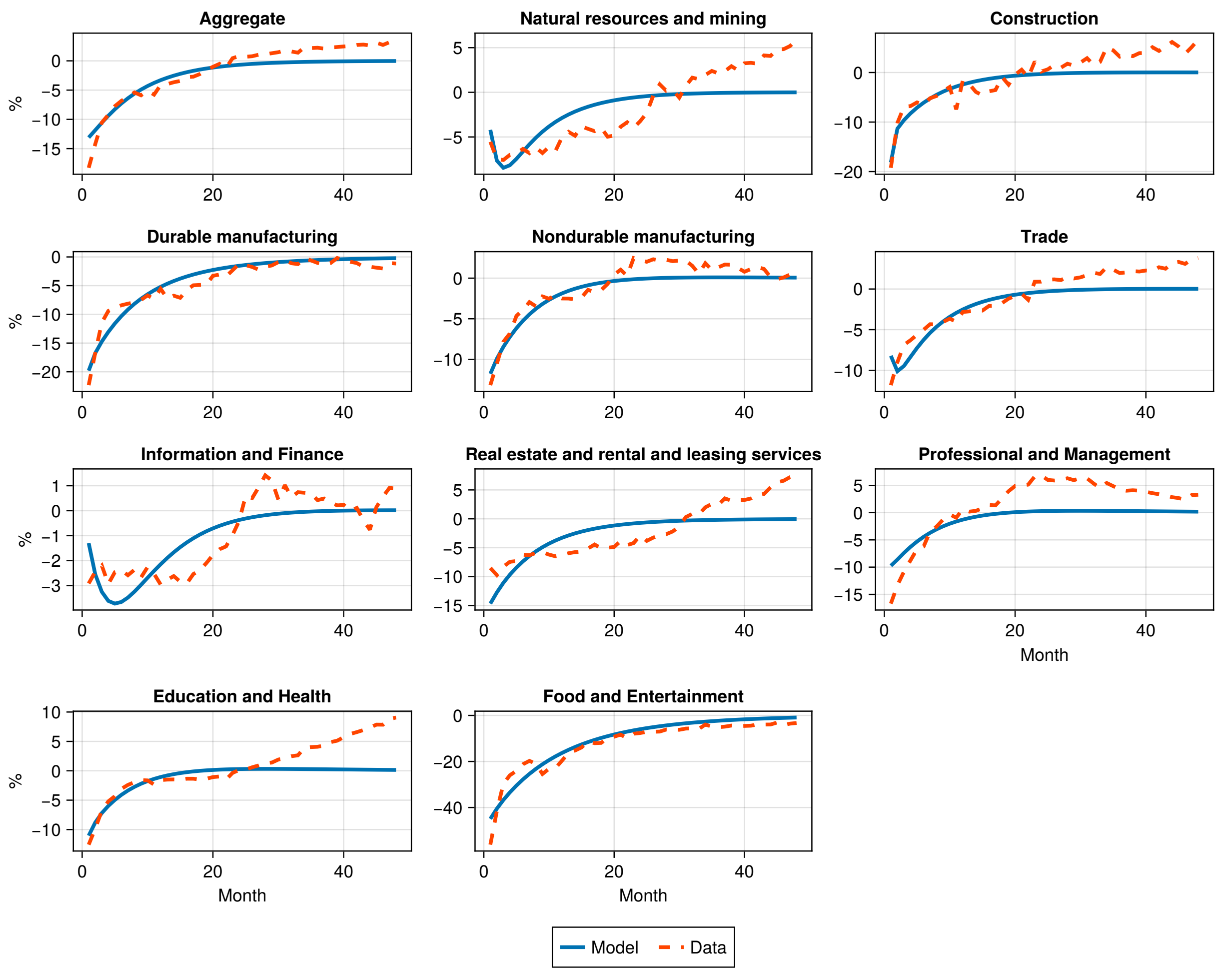
Among Natural resources and Mining, and Information and Finance, the model fails to match the prolong periods of negative responses at the start of COVID-19. Instead, the impulse response functions exhibit "U-shapes" during those periods. Without frictions in the labor market or adjustment cost in capital or occupational choice, the current model lacks a mechanism to generate a

[6]A possible justification is that the government mainly borrows from foreign entities, and the domestic tax payers do not take into consideration the repayment in the future.

[7]A rough comparison is: there are over 400 data points, 10 sectors with more than 40 months of data, and 20 parameters with regularity conditions.

persistent decline after shocks. Intuitively, the model artificially creates excessively large declines then faster recovery to match the "means" of the transition paths in the data. Additionally, because the lack of growth, the model is unsuited to match the sustained developments happened two years after COVID-19, as in Education and Health, and Construction.

Figure 13: Fit of the Labor Impulse Response Function with Realized Shock



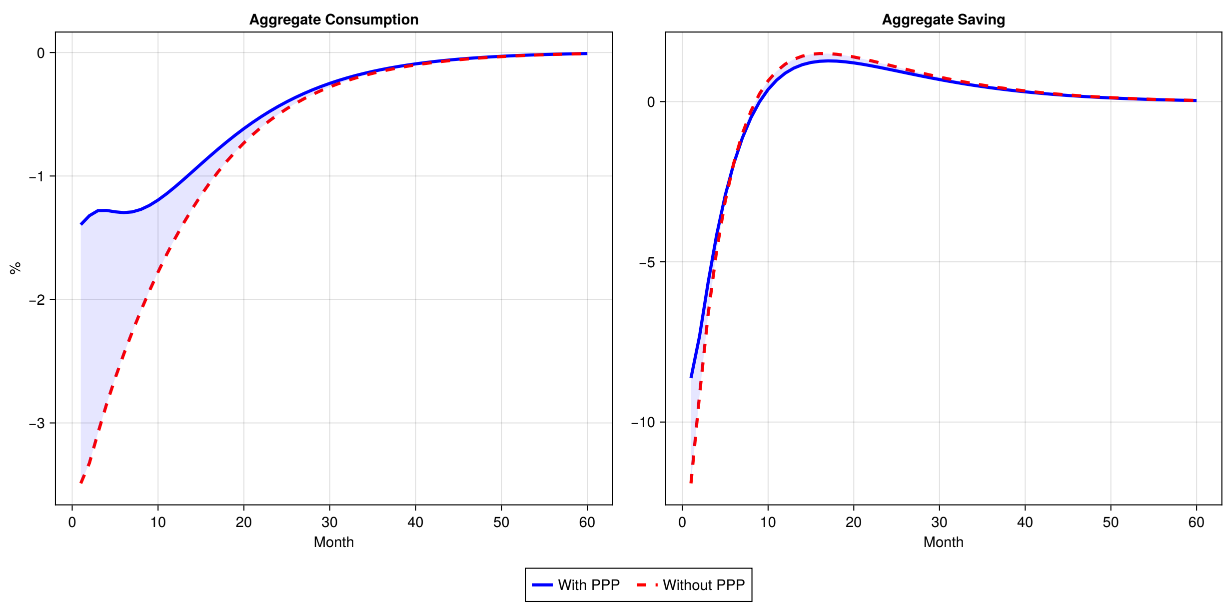
This figure shows the impulse response function fits of the labor input dynamics after COVID-19. Red lines are time series from data. Blue lines are impulse response functions from the model after the *realized shocks*, $\{\Psi_t\}$. x-axis is the the number of months after COVID-19. y-axis is the percentage change from the pre-COVID-19/Steady-state value.

4.4 Model Generated Responses

The red dotted lines in Figure 14 shows the first order responses of aggregate variables with respect to the *pure COVID-19 Shock*s, i.e. without the PPP. Aggregate consumption falls by 3.5% on impact. It takes around 2 years to recover to a level less than 0.5% lower than the steady-state.

Aggregate saving falls by 12% on impact. The initial decline in aggregate capital stock is then offset by a period of excessive saving, comparing to steady-state saving rate, starting around 1-year after the initial shock. The blue lines show the responses with *realized shocks*, which corresponds to what actually happened in the data, i.e. with the PPP. Consumption and saving both fall less. Particularly, aggregate consumption has a 2 percentage points improvement, while aggregate saving has a 4 percentage points gain. For welfare analysis, the shaded areas between the two curves denote the cumulative effects of the PPP on the aggregate variables. In a counterfactual world without the PPP, aggregate consumption would have declined by 22% over a five-year horizon[8]. With the PPP, the impact has been mitigated to 12%. To put this number into perspective, with a cost of 4% GDP in 2019, the PPP is able to generate a 10 percentage points gain in aggregate consumption after COVID-19.

Figure 14: IRFs of Aggregate Consumption and Saving after COVID-19



This figure shows the impulse response functions in aggregate consumption (left) and aggregate saving (right) during COVID-19 in the model. Blue solid lines are the IRFs with respect to the *realized shocks*, i.e. with the PPP. Red dashed lines are the IRFs with respect to the *pure COVID-19 shocks*, i.e. without the PPP. X-axis is the number of months. Y-axis is the percentage change relative to the steady-state value.

Turn to sectoral firm dynamics, Figure 15 plots the percentage changes in the numbers of entrepreneurs (firms) after COVID-19 in the model (blue and red) and in the data (green). Most of

[8]After accounting for household time preference. That is

$$\int e^{-\rho t} \Delta C dt.$$

the sectors experience increases in the number of businesses after COVID-19. This is consistent with the increase in new business application documented by Fazio et al. (2021) and Decker and Haltiwanger (2023). Decker and Haltiwanger (2023) finds that after the initial impact, there have been increases in the numbers of openings in almost all sectors, particularly in Construction, Retail Trade, Transportation, and Health Care. My model is able to reconcile these facts.

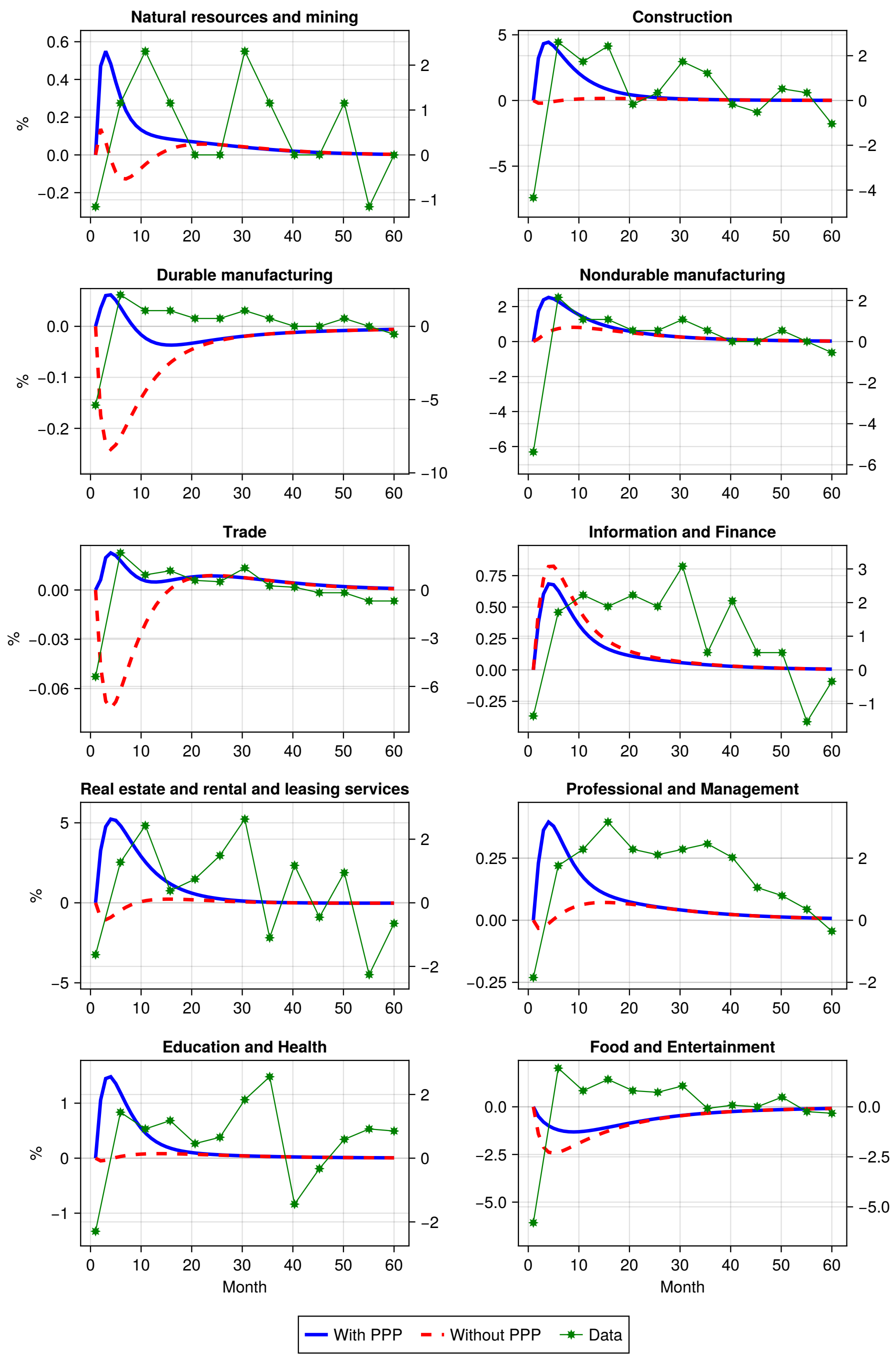
In Figure 15, the data show an uniform decline in the number of firms in response to COVID-19. This is in line with the labor demand statistics shown in Figure 11. However, firm dynamics show quick and long-lasting rebound after the shock. This true for all sectors but with different degrees. In the model, this phenomenon is due to the support from the firm subsidy program. The PPP is able to generate strong firm entries despite the negative shocks to aggregate labor supply and preferences. In particular, for Construction, Manufacturing, and Trade, the impulse responses functions with respect to the *realized shocks* match the inverted "U-shaped" patterns in the data closely.

In the model, extra wealth relaxes the financial constraint. Entrepreneurs who would have closed their businesses due to limited operation scales can keep their firms open. Moreover, the shifts in household preferences disfavor Food and Entertainment because of the high contact-intensity nature of the sector. This increases the profits of other sectors, leading to more entries.

One concern regarding the PPP is the possibility of "subsidy fraud". This means creating a "fake" business just to game the subsidies. Autor et al. (2022a) and Autor et al. (2022b) find evidence against this concern. They find that the PPP actually reaches real businesses and supports their operation expenses. On the other hand, because entrepreneurs cannot earn wage at the same time, the notion of "fake entrepreneur" does not have a clear counterpart in the model. In fact, since the PPP is modeled as a gradual rollout, this means entrepreneurs who enter because of the PPP are likely to stay. The inefficiency comes from another source: the PPP not only helps productive entrepreneurs to keep their businesses, it also introduces unproductive entrepreneurs, who would have been workers absent the subsidy. The gain on aggregate productivity thus depends on the balance of these two forces. From the effect on aggregate consumption, as in Figure 14, we can conclude that the net effect is productivity enhancing.

Due to the lack of growth in the model, the impulse responses converge to zero as the model returns to steady state. This limits the model's ability to capture the later stages of the series, where sectors such as Information, Finance, and Professional Management show sustained growth in firm numbers. This growth may reflect the relaxed monetary environment following the pandemic. An extension with explicit monetary policy changes is open for future research.

Figure 15: IRF of Sectoral Number of Entrepreneurs after COVID-19



This figure shows the responses in sectoral numbers of entrepreneurs during COVID-19 in the model. X-axis is the number of months from period 0, when the shock happens. Y-axis is the percentage change relative to the steady-state value. Blue solid lines are the IRFs with *realized shocks*. Red dashed lines are the IRFs with *COVID-19 shocks*. Green lines are quarterly data series of the net number of openings from BED, normalized by standard deviation. For information related to the data, refer to Appendix C.

4.5 Redesigning the Paycheck Protection Program

In this section, I explore possible improvements over the PPP by varying the distribution of funds while keeping its original size and rollout speed unchanged. This numerical exercise takes the following steps: Choose a criterion to evaluate the performance of a counterfactual PPP, e.g. aggregate consumption. Assume a linear functional form for the shares with three factors: the severity of impact, the extent of financial constraint, and the centrality within the network. Finally, search on the loadings of these factors for a scheme that maximizes the criterion, conditional on the realization of *COVID-19 shocks*.

From the exercises in Section 2.1 and Section 2.2, different subsidy timing can have different outcomes depending on the pre-existing equilibrium distribution. However, by holding constant the size and rollout speed, I isolate the channel associated with sectoral differences and production network. The goal is to see what the government should have done, if taking the network structure and sectoral differences into consideration.

I explore two criterions: aggregate consumption and aggregate output. Both objects are the cumulated sum of discounted values along the transition path. Graphically, they are represented by the area under the blue curves in Figure 14[9]. Aggregate output is clearly defined in the model as the sum of sectoral outputs. The corresponding object in the data is GDP. Improvements in aggregate output can come from two channels: (1) less resource misallocation from frictions within the network, (2) less distortion on households' intertemporal saving decisions. Aggregate consumption is closer to a welfare measure. By this metric, a good subsidy program should not only help the firms under financial stress, but also provide support to sectors that households prefer. Although changes in household preferences are unlikely to be directly observed by the government, they can be partially inferred from the varying severity of responses across sectors.

The distribution of fund, $\mathbf{\Pi}$, is assumed to have a linear structure with three factors: sectoral change in labor input, $\mathbf{\Delta L}$, a sectoral measure of financially constrained firms[10], $\mathbf{\Delta B}$, and sectors' network centrality, $\mathbf{C}$:

$$\mathbf{\Pi} = \alpha_0 + \alpha_1 \mathbf{\Delta L} + \alpha_2 \mathbf{\Delta B} + \alpha_3 \mathbf{C}. \tag{4.2}$$

In the numerical exercise, I allow the policy maker to have access to the average value of the independent variables in the first 2 periods after shocks. In other words, $\{\overline{\mathbf{\Delta L}}, \overline{\mathbf{\Delta B}}, \overline{\mathbf{C}}\}$ is the information set of the government. The search for optimal policies is on the parameters $\{\alpha\}$ given a criterion. Table 2 shows the fund distribution between sectors for the PPP and counterfactual policies.

[9]Since the responses to aggregate consumption and output are negative, the maximization means minimizing the absolute value of the areas between horizontal axis and response functions.

[10]In the model, this is clearly defined as the number of firms facing a binding constraint. In the data, this can be observed from firm loan applications, or firm surveys.

Figure 16 shows the effects of two counterfactual policies. The responses without the PPP (red) are treated as baselines. The rest of the lines plot the impulse response functions as the difference to the baseline. Blue lines plot the original PPP, as in Figure 14. Green lines plot the counterfactual program that maximizes discounted aggregate consumption. Orange lines plot the program that maximizes aggregate output (GDP). First observation from these figures is that redesigning fund distribution leads to a modest improvement over the original PPP. Integrating the area between the green and blue lines, adjusting for discounting, the redesigned PPP yields an 1 percentage point increase in aggregate consumption on top of the 10 percentage points gain from the original PPP. Put in other words, if the PPP was designed in the way that maximizes its impact on aggregate consumption, it takes $95 billion less to achieve the same result on consumption support. This difference is more substantial for aggregate output. According to Figure 16, the original PPP increases aggregate output by 1.4 percentage points. By redesigning with a focus on GDP, it can increase aggregate output by 3 percentage point in total[11].

Table 2: Redesign Policy Weights

	Actual PPP	**Consumption**	**GDP**
Natural resources and mining	0.03	0.0	0.18
Construction	0.16	0.15	0.72
Durable manufacturing	0.08	0.29	0.02
Nondurable manufacturing	0.03	0.17	0.07
Trade	0.14	0.0	0.0
Information and Finance	0.04	0.0	0.0
Real estate and rental and leasing services	0.04	0.19	0.0
Professional and Management	0.13	0.06	0.0
Education and Health	0.19	0.14	0.01
Food and Entertainment	0.16	0.0	0.0

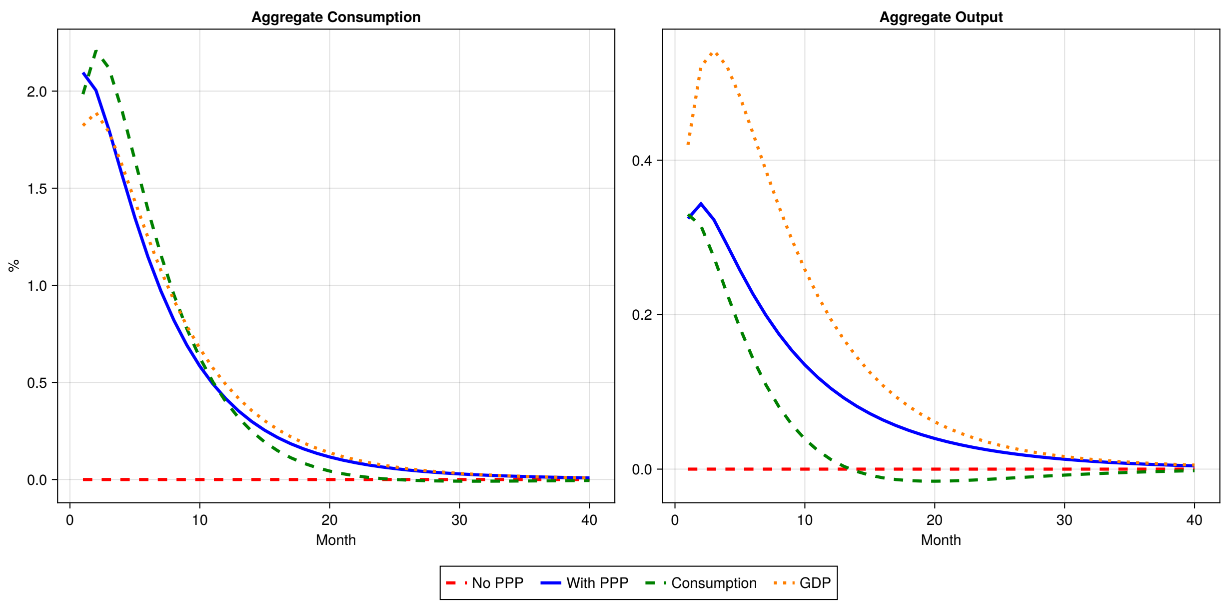
This table reports the optimal weights across sectors resulting from the linear structure in Equation 4.2. The first column reports the weights of the actual PPP, the second column reports the optimal weights that maximizes aggregate consumption. The third column reports the optimal weights that maximizes aggregate output.

Some explanations are due for these results. In the model, the PPP increases the wealth of entrepreneurs directly. This relaxes the financial constraints of both productive and unproductive entrepreneurs. More directly, it increases the aggregate capital supply in the economy. However, this does not translate to an increase in aggregate output one-to-one. First, production technologies in all sectors are decreasing return-to-scale. Second, unlike capital supply, labor supply in this setup is bounded. For example, in a frictionless environment, labor supply is fixed as the set of entrepreneurs is fixed. Lastly, the productivity gain from relaxation of financial constraints depends

11

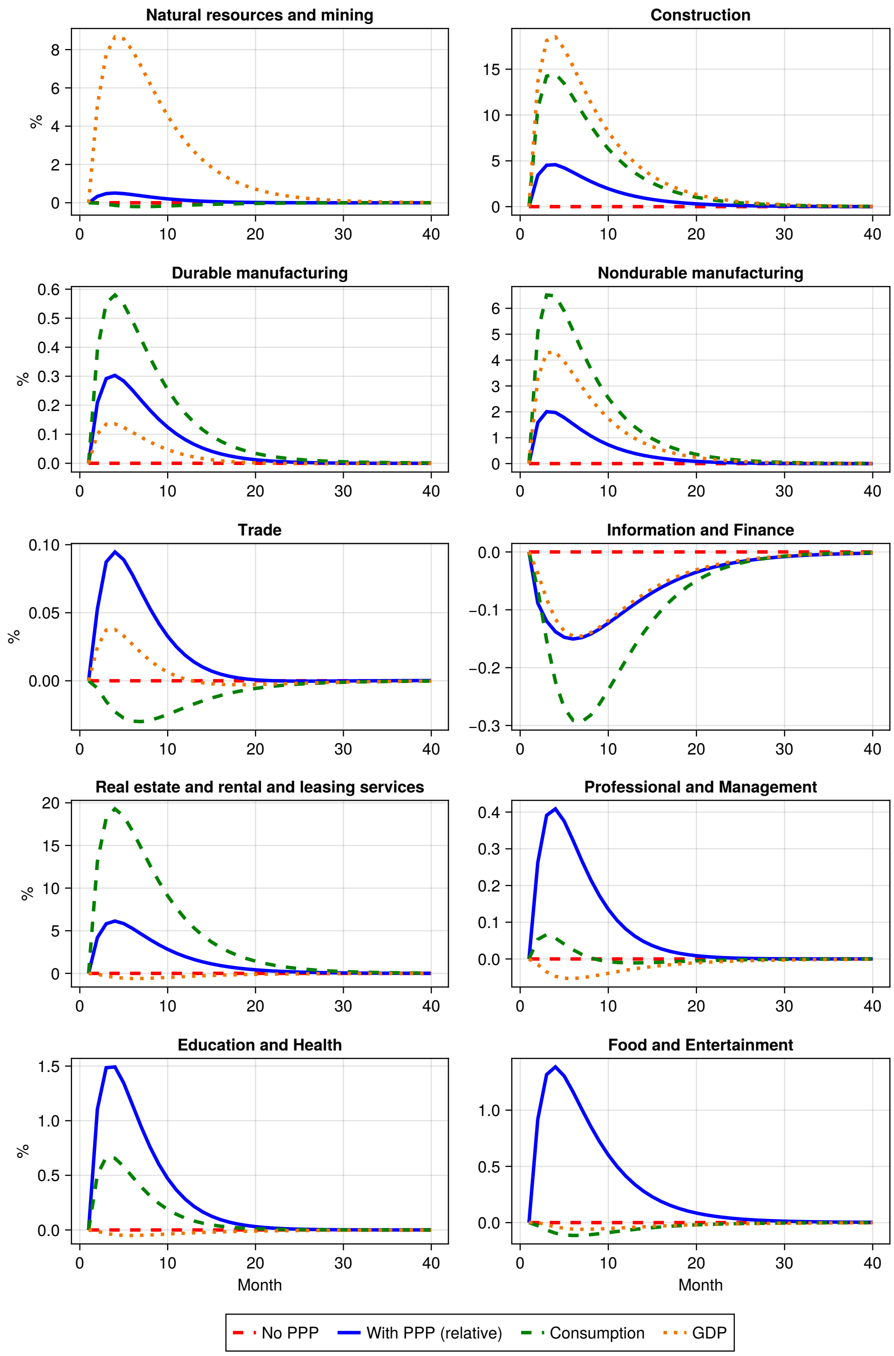
on the joint distribution of idiosyncratic productivity and wealth among entrepreneurs. The PPP is assumed to target sectors rather than to target financially stressed firms because those firms are hard to identify in practice and the targets have to rely on self-reporting. This creates a gap in first order efficiency between the PPP considered here and the program that can target a subset of firms directly. Considering these restrictions, the fact that the original PPP can generate sizable gain by improving the financial condition of entrepreneurs suggests a substantial efficiency loss stemming from sector-specific financial frictions. The notable increase in aggregate output with the redesigned PPP highlights the inefficiencies in the initial allocation of funds, which did not consider the impact of the production network.

Figure 16: Counterfactual Policies - Aggregate Outcomes



This figure shows the impulse response functions (IRFs) from the model, with different implementations of the PPP, using No PPP as the baseline. The red dashed lines, overlapping with the x-axis, are the IRFs with respect to a counterfactual scenario where the PPP was not implemented. The blue lines are the IRFs with respect to the actual PPP implementation. The green lines are the IRFs with respect to a counterfactual PPP implementation that maximizes aggregate consumption. The orange lines are the IRFs with respect to a counterfactual PPP implementation that maximizes aggregate output. On the left panel is for aggregate consumption, the right panel is for aggregate output. The x-axis is month. The y-axis is percentage change from the pre-COVID-19 level.

Figure 17: Counterfactual Policies - Sectoral Firm Dynamics



This figure shows the model's impulse response functions (IRFs) of the numbers of firms for each sectors, with different implementations of the PPP, using No PPP as the baseline. The red dashed lines, overlapping with the x-axis, are the IRFs with respect to a counterfactual scenario where the PPP was not implemented. The blue lines are the IRFs with respect to the actual PPP implementation. The green lines are the IRFs with respect to a counterfactual PPP implementation that maximizes aggregate consumption. The orange lines are the IRFs with respect to a counterfactual PPP implementation that maximizes aggregate output. The sector names are included as subtitles. X-axis is month. Y-axis is percentage change from the pre-COVID-19 level.

Figure 17 plots the firm dynamics by sector with the counterfactual policies. The line styles follows the ones in Figure 16. What stands out is the differences in firm dynamics between upstream and downstream sectors. With a consumption maximizing program, entry in Food and Entertainment, Education and Health, and Trade is greatly reduced in exchange for more entry in Manufacturing and Real Estate. This is due to the change in household preferences. To maximizes aggregate consumption, the optimal policy shifts away from the sectors households dislike due to the demand shocks. In the case of COVID-19, this means contact-intensive sectors like Food and Entertainment. A output maximizing program puts more weights on upstream sectors because the stimulant effect can be propagated to other sectors through the production network. As a result, there are more firm entries in Natural Resources, Construction, and Manufacturing.

Comparing to data on firm dynamics in Figure 15, redesigning the PPP attenuates the large numbers of entries in downstream sectors. In particular, the increases in the number of firms in Professional and Management and Food and Entertainment fall from 1.5% and 0.4% to negligible levels respectively. This means the increases in firm entry among these sectors, as documented by Decker and Haltiwanger (2023), can be partially attributed to the firm subsidy programs during this period. However, this does not have clear welfare implications. In the model, the labor market is frictionless, so entrepreneurs who are driven out of business can immediately find jobs. As a result, there is no unemployment and the large welfare loss associated with job destruction and frictional search process is omitted. In fact, by moving away from sectors that households dislike after the preference shocks, aggregate outcomes can improve due to increased labor supply for other sectors. The results regarding firm dynamics remain valuable nonetheless because such subsidy programs are often designed with intention to prevent massive firm closures.

5 Conclusion

This paper presents a novel quantitative framework that can be used to assess the effectiveness of direct firm subsidy programs, while accounting for the network structure underlying the production economy. In additional to the classical models in production network literature, my framework incorporates financial frictions, capital accumulation, and entrepreneurship, making the model dynamic on the household side. These features allow me to study the implications of subsidies on sectoral firm dynamics, which is a central aim of such programs.

The quantitative performance of this model depends on the calibration of sectoral levels of financial frictions and production technologies. On this front, I estimate these parameters via SMM to match established reduced-form evidences from the empirical corporate finance literature. The Paycheck Protection Program (PPP) reduces the cumulative impact of COVID-19 on aggregate consumption from -22% to -12% over a 5-year span and supports the observed increase in new business formation. Modifying the PPP by reallocating funds across sectors results in a 1 percentage point increase in aggregate consumption and mitigates excessive firm creation in downstream sectors.

In conclusion, this paper underscores the importance of production networks in designing subsidy programs during crises. Although these policies are often time-sensitive, using pre-shock sectoral linkages can enhance aggregate outcomes and yield significant sectoral effects.

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A A Two Period Model

Time is discrete. There are two periods: $t = 0, 1$. There is a continuum of households with measure $\mathcal{S}$. The households are ex-ante heterogeneous in terms of the technology they are born with. The households are equally divided into S sectors, with corresponding production function. In period 0, households receive endowments in the form of final consumption good and decide how much to consume. The household can save by transforming final consumption good into capital. Their idiosyncratic productivities $z_{i,0}$ are also revealed. In period 1, households receive income, in the form of wage or profit, depending on occupational choice, the prices of factor inputs, and the prices of sectoral goods and final goods.

The household solves the following problem

$$\max_{C_{i,0},e} u\left(C_{i,0}\right) + \rho \mathbb{E}_0\left[u\left(C_{i,1}\right)\right]$$
$$a = \omega - C_{i,0}$$
$$e\pi\left(z,a\right) + \left(1-e\right)w + ra = P_1 C_{i,1}$$

In other words, the household chooses how much to save in the first period. In the second period, the household chooses the occupation. Throughout this section, I normalize the price of the final consumption goods, P_1 to 1.

The Worker supplies labor inelastically, and earns w. The entrepreneurial decision is made based on

$$w \lesseqgtr \pi_{i,s}\left(z,a\right)$$

The production side is intratemporal and follows the structure in the infinite horizon case. In a world without financial friction, the occupational choice only depends on the realized productivity at $t = 1$. With financial friction, the profit depends on whether the entrepreneur can operate at the optimal scale, thus affecting the occupational choice.

A.1 Equilibrium Properties

Definition 2. Competitive Equilibrium

A competitive equilibrium in this environment is the collection of prices $\left\{r, w, \left\{P_s\right\}_{s\in\mathcal{S}}\right\}$, and household actions $\left\{C_{i,0}, e_{i,s}, k_{i,s}, l_{i,s}, x_{i,s,h}\right\}_{i\in\mathcal{I},s,h\in\mathcal{S}}$, sectoral aggregator and final aggregator's action $\left\{\left\{q_{i,s}\right\}_{i\in\mathcal{I}_s,s\in\mathcal{S}}, \left\{C_s\right\}_{s\in\mathcal{S}}\right\}$ such that given initial condition at $t = 0$, $\left\{\omega_i, z_{i,0}\right\}_{i\in\mathcal{I}}$, and the stochastic process of idiosyncratic productivity shock, $\Psi\left(z_{i,1}|z_{i,0}\right)$, household's actions solve the household's utility maximization problem, aggregator's actions solve their profit maximization problem. The actions induce the distribution of productivity and wealth, $G\left(z,a\right)$ at $t = 1$. Finally, markets clear at $t = 1$

1. Final goods market clears
$$\int C_{i,t} dG(a,z) = C_1$$

2. Sectoral goods markets clear
$$Q_s = C_s + \sum_{h \in \mathcal{S}} \left[\int x_{i,h,s} \mathbf{1}_{\{i \in \mathcal{E}_h\}} dG_s(a,z) \right], \forall s \in \mathcal{S}$$

3. Firm's goods markets clear
$$q_{i,s} = q^d_{i,s}, \forall s \in \mathcal{S}$$

4. Capital market clears
$$\sum_{s \in \mathcal{S}} \int k_{i,s,t} \mathbf{1}_{\{i \in \mathcal{E}_s\}} dG_s(a,z) = \int a_{i,t} dG(a,z)$$

5. Labor market clears
$$\sum_{s \in \mathcal{S}} \int l_{i,s,t} \mathbf{1}_{\{i \in \mathcal{E}_s\}} dG_s(a,z) = \int \mathbf{1}_{\{i \notin \mathcal{E}\}} dG(a,z)$$

   where $\mathcal{E}$ denotes the set of entrepreneurs, and $\mathcal{E}_s$ denotes the set of entrepreneurs in sector s.

In this two-period model, the saving decision of the households can be characterized in closed form because there is no continuation value afterwards. This allow me to study the implication of financial friction and production network on aggregate saving and consumption. Before stating the main results, the following lemmas show how individuals behaviors can be aggregated to the sectoral level, and applied to the production network tools in the literature.

Lemma 1. *Sectoral Aggregation*

Without financial friction, the aggregate output from sector s, Q_s, can be expressed as

$$Q_s = Z_s (K_s)^{\alpha_s} (L_s)^{\beta_s} \left(\prod_{h \in S} (X_{s,h})^{g_{s,h}} \right)^{\gamma_s} \tag{A.1}$$

$$Z_s = \left[\int_0^{N_s} z_{i,s}^{\frac{1}{1-(\alpha_s+\beta_s+\gamma_s)}} di \right]^{1-(\alpha_s+\beta_s+\gamma_s)} \tag{A.2}$$

$$K_s = \int_{\bar{z}_s}^{\infty} k(z) \, d\Psi_s(z), L_s = \int_{\bar{z}_s}^{\infty} l(z) \, d\Psi_s(z), X_{s,h} = \int_{\bar{z}_s}^{\infty} x_h(z) \, d\Psi_s(z), h \in \mathcal{S}$$

With financial friction,

$$Q_s = \tilde{Z}_s (K_s)^{\alpha_s} (L_s)^{\beta_s} \left(\prod_{h \in S} (X_{s,h})^{g_{s,h}} \right)^{\gamma_s} \tag{A.3}$$

$$\tilde{Z}_s = \left[\int_{\mathcal{E}_s} \left(z_{i,s} / \phi_{i,s}^{\alpha_s} \right)^{\frac{1}{1-(\alpha_s+\beta_s+\gamma_s)}} dG_s(z,a) \right]^{1-(\alpha_s+\beta_s+\gamma_s)} \tag{A.4}$$

$$\phi_{i,s} = \frac{r + \mu_{i,s}}{r}$$

$$K_s = \int_{\mathcal{E}_s} k(z,a)\, dG_s(z,a), L_s = \int_{\mathcal{E}_s} l(z,a)\, dG_s(z,a), X_{s,h} = \int_{\mathcal{E}_s} x_h(z)\, dG_s(z,a)$$

N_s is the number of firms in sector s, Z_s is the sectoral aggregate productivity, Ψ_s is the CDF of idiosyncratic productivities in sector s, $\bar{z}_s$ is the cutoff value on productivity that determines the occupational choice, K_s, L_s, $X_{s,h}$ are aggregate capital input, labor input, and intermediate good input of sector s respectively. $\mu_{i,s}$ is the Lagrangian multiplier on the borrowing constraint in entrepreneur's profit maximization problem.

Lemma 1 shows that the functional form of the individual production function is inherited at the sectoral level even with arbitrary production network. The sectoral level aggregate productivity is a geometric sum of the productivities of active entrepreneurs. Comparing (A.1) with (A.3), financial friction affects the effective sectoral productivity $\tilde{Z}_s$ with $1/\phi_{i,s}^{\alpha_s}$ creating a weighting scheme on firm productivities. Financially constrained firms have $\phi_{i,s} > 1$ thus decreasing its weight in the aggregation. This is because the they have to operate at a suboptimal scale, therefore harm productivities. This part corresponds to majority of the work in the production network literature (Acemoglu et al., 2012; Liu, 2019; Baqaee and Farhi, 2020; Bigio and La'O, 2020). What is different is that often a constant return-to-scale (CRS) technology is assumed with CES demand system. The CES demand system generates misallocation within and across sectors by itself. To isolate the effect of financial friction, I adopt the decreasing return-to-scale (DRS) assumption as in Buera, Kaboski, and Shin (2011) and derive clean aggregation expressions as in the CRS case.

The occupational choice without financial friction reduces to a cut-off level of productivity for each sector, $\bar{z}_s$. With financial friction, occupational choice depends on the joint distribution of household wealth a and productivity z within the sector, $G_s(a,z)$. This is because wealth enters the profit function of the entrepreneur through borrowing limit. Productive yet poor households may choose not to start their own firm because they cannot operate at the optimal scale. There is no analytical expression for the total mass of entrepreneurs in each sector, unless we only consider the case without financial friction and assume a particular idiosyncratic productivity process[12].

[12] See Buera, Kaboski, and Shin (2011) for a special case in their environment.

Lemma 2. *Sectoral Optimality Condition*

At the sectoral level, the first order condition with respect to capital, labor and intermediate goods can be expressed as

$$\alpha_s \frac{Q_s}{\phi_s^K K_s} = \frac{r}{P_s} \tag{A.5}$$

$$\beta_s \frac{Q_s}{L_s} = \frac{w}{P_s} \tag{A.6}$$

$$\gamma_s g_{s,h} \frac{Q_s}{X_{s,h}} = \frac{P_h}{P_s} \tag{A.7}$$

where

$$\phi_s^K = \frac{\int_{\mathcal{E}_s} \phi_{i,s} k_{i,s} dG_s}{K_s}$$

In the frictionless world, $\phi_s^K = 1, \forall s$.

Lemma 2 follows from the production function and the first order conditions of individual firms. Because $\phi_{i,s} \geq 1, \forall i \in \mathcal{I}_s, \forall s$, $\phi_s^K \geq 1, \forall s$. This mechanically means with financial friction, at a sector level, the marginal product of capital is higher than the cost of capital rental. The effect of this distortion depends on the correlation between the presence of financial friction and firm size. If the firms which are supposed to be big are financially constrained, the distortion on marginal decision is larger. It becomes useful in deriving the following propositions.

Proposition 1. *Aggregate Saving*

Without financial friction, the aggregate savings at $t = 0$*,*

$$K = \rho \boldsymbol{\alpha}' diag \left(\frac{C_1 \int_{i \in \mathcal{I}_s} \mathbb{E}_0 \left[U_c \left(C_{i,1} \right) \right] dG_s}{\int_{i \in \mathcal{I}_s} U_c \left(C_{i,0} \right) dG_s} \right) \left\{ \mathbb{I}_S - \left(diag \left(\boldsymbol{\gamma} \right) \mathbf{G} \right)' \right\}^{-1} \boldsymbol{\nu} \tag{A.8}$$

With financial friction, the aggregate savings at $t = 0$*,*

$$K = \rho \boldsymbol{\alpha}' diag \left(\frac{C_1 \mathbb{E}_0 \left[\int_{i \in \mathcal{I}_s} U_c \left(C_{i,1} \right) \left(1 + \theta_s \frac{\mu_{i,s}}{R} \right) dG_s \right]}{\int_{i \in \mathcal{I}_s} U_c \left(C_{i,0} \right) dG_s} \right) \left[diag \left(\boldsymbol{\phi}^K \right) \right]^{-1} \left\{ \mathbb{I}_S - \left(diag \left(\boldsymbol{\gamma} \right) \mathbf{G} \right)' \right\}^{-1} \boldsymbol{\nu} \tag{A.9}$$

Comparing (A.8) and (A.9), we can see that two forces emerge from financial friction working against each other to affect the level of saving by the households in aggregate. $\boldsymbol{\phi}^K$ measure the levels of distortion on marginal productivity of all sectors. Higher distortion means lower capital demand at $t = 1$. $\theta_s \frac{\mu_{i,s}}{R}$ measures the extra value of wealth in household's Euler Equation. This is because wealth can not only earn interest rate at $t = 1$, but can also act like collaterals when needed.

The term $\left\{\mathbb{I}_S - (\text{diag}(\boldsymbol{\gamma})\,\mathbf{G})'\right\}^{-1}$ shows that financial friction within a sector is propagated via the network. Its effect on aggregate savings depends on its centrality measure.

The term $\mathbb{E}_0\left[\int_{i\in\mathcal{I}_s} U_c\left(C_{i,1}\right)\left(1+\theta_s\frac{\mu_{i,s}}{R}\right)dG_s\right]$ summarizes the importance of heterogeneity among households even in the same sector. When households in sector s are identical in productivity, z_0, but heterogeneous in endowment ω_i, richer households have lower $U_c\left(C_{i,1}\right)$ and lower $\theta_s\frac{\mu_{i,s}}{R}$ because they are less likely to be constrained, holding the same transition probability of technology. This means inequality in terms of wealth endowment increases saving. When households are identical in endowment ω, but heterogeneous in initial productivities, and further assume productivity is slow-moving, for more productive households, $U_c\left(C_{i,1}\right)$ will be lower and $\theta_s\frac{\mu_{i,s}}{R}$ will be higher because they would like to operate at bigger scales. This means dispersion in productivity does not induce more savings.

Proposition 2. *Aggregate Consumption*

Without financial friction, the aggregate consumption at $t=1$ in logarithm is

$$\log C_1 = \boldsymbol{\nu}'\mathbf{L}\left\{\begin{array}{c}\log\mathbf{Z}+\boldsymbol{\alpha}\log K+\boldsymbol{\beta}\log L-\boldsymbol{\beta}\log\Lambda\\ -\boldsymbol{\alpha}\log\left(\rho\boldsymbol{\alpha}'\left\{\mathbb{I}_S-(diag\left(\boldsymbol{\gamma}\right)\mathbf{G})'\right\}^{-1}\boldsymbol{\nu}\right)+\tilde{\mathbf{D}}\end{array}\right\} \tag{A.10}$$

where

$$\Lambda = \mathbf{1}'diag\left(\boldsymbol{\beta}\right)\left\{\mathbb{I}_S-(diag\left(\boldsymbol{\gamma}\right)\mathbf{G})'\right\}^{-1}\boldsymbol{\nu}$$

$\mathbf{L}=\left(\mathbb{I}_\mathbf{S}-diag\left(\boldsymbol{\gamma}\right)\mathbf{G}\right)^{-\mathbf{1}}$ *is the Leontief Inverse Matrix.* $\tilde{\mathbf{D}}$ *is a constant that only depends on model parameters.*

With financial friction, the aggregate consumption at $t=1$ in logarithm is

$$\log C_1 = \boldsymbol{\nu}'\mathbf{L}\left\{\begin{array}{c}\log\tilde{\mathbf{Z}}+\boldsymbol{\alpha}\log K+\boldsymbol{\beta}\log L-\boldsymbol{\alpha}\log\boldsymbol{\phi}^K-\boldsymbol{\beta}\log\Lambda\\ -\boldsymbol{\alpha}\log\left(\rho\boldsymbol{\alpha}'\left[diag\left(\boldsymbol{\phi}^K\right)\right]^{-1}\left\{\mathbb{I}_S-(diag\left(\boldsymbol{\gamma}\right)\mathbf{G})'\right\}^{-1}\boldsymbol{\nu}\right)+\tilde{\mathbf{D}}\end{array}\right\} \tag{A.11}$$

Proposition 2 shows the decomposition of aggregate consumption. Intuitively, $\log\mathbf{Z}+\boldsymbol{\alpha}\log K+\boldsymbol{\beta}\log L$ resembles the log individual production functions in each sectors. The constant Λ reflects the general equilibrium forces that determine the supply of labor. In particular, household's budget constraint links consumption to labor income, and production network transfers the labor supply to final consumption goods. $\boldsymbol{\alpha}\log\left(\rho\boldsymbol{\alpha}'\left\{\mathbb{I}_S-(\text{diag}\left(\boldsymbol{\gamma}\right)\mathbf{G})'\right\}^{-1}\boldsymbol{\nu}\right)$ is the intertemporal forces from Proposition 1 that determines the supply of capital into the second period.

The goal of Proposition 2 is not to compute the level of aggregate consumption, but to have a notion on aggregate productivity. Comparing the terms in (A.10) and (A.11), financial friction affects the aggregate productivity through both the intensive margin and extensive margin. The extensive margin is reflected in the construction of sectoral productivities in Lemma 1. Financial friction distort occupational choice decisions by the households. This determines which part of the distribution enters the aggregate productivity $\tilde{Z}_s$, as well as the amount of labor supply into the economy. The intensive margin shows up explicitly in the term $\log \boldsymbol{\phi}^K$ in A.11 as well as in A.4. In particular,

$$\log \phi_s^K = \log\left(\int_{\mathcal{E}_s} \phi_{i,s} k_{i,s} dG_s\right) - \log\left(\int_{\mathcal{E}_s} k_{i,s} dG_s\right) \geq 0.$$

Since $k_{i,t}$ is positively related to idiosyncratic productivity, financial friction has larger impact if it is the more productive firms that are financially constrained. In a setting with occupational choice, this is indeed the case. This is because the households who make occupational choice on the marginal can be divided into two groups: productive but poor, unproductive but wealthy. By definition, the second group is unaffected by borrowing constraint. whereas the first group contributes positively to the expression above.

B Sequence-space Jacobian Method

In this section, I describe how to apply the sequence-space Jacobian method by Auclert et al. (2021) to solve the transition dynamics around steady state.

The general equilibrium of the model at time t is characterized by the following system of equations

$$\mathbf{F_t}(\mathbf{X},\mathbf{Z}) = \begin{pmatrix} K^D(\mathbf{X},\mathbf{Z}) - K^S(\mathbf{X},\mathbf{Z}) \\ L^D(\mathbf{X},\mathbf{Z}) - L^S(\mathbf{X},\mathbf{Z}) \\ Q_1^D(\mathbf{X},\mathbf{Z}) - Q_1^S(\mathbf{X},\mathbf{Z}) \\ \vdots \\ Q_s^D(\mathbf{X},\mathbf{Z}) - Q_s^S(\mathbf{X},\mathbf{Z}) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

The exogenous variable set $\mathbf{Z} = \{O_t, \boldsymbol{\nu}_t, \dots\}$ contains the shocks to aggregate labor supply, O_t, the shocks to household preferences on the final consumption bundle, $\{\boldsymbol{\nu}_t\}$, the PPP series, $\{\mathcal{P}_t\}$, and potentially other kinds of shocks. The endogenous variable set $\mathbf{X}_t = \{r_t, w_t, \mathbf{P}_t\}$ contains the interest rate r_t, wage, w_t, and prices of sectoral goods, $\mathbf{P}_t$. $F_t(X,Z)$ contains the capital market clearing condition, the labor market clearing condition, and the sectoral goods market clearing conditions. By the implicit function theorem,

$$d\mathbf{X}_t = -\mathbf{F}_{\mathbf{X},t}^{-1}\mathbf{F}_{\mathbf{Z},t} d\mathbf{Z}_t \equiv \mathbf{G}_t d\mathbf{Z}_t$$

In this representation, $\mathbf{F_t}(\mathbf{X},\mathbf{Z})$ has size 12. $\mathbf{F}_{\mathbf{X},t}: 12 \times 12, \mathbf{F}_{\mathbf{Z},t}: 12 \times 2$.

We can use a more compact representation to characterize the equilibrium conditions along the whole time path.

$$\mathbf{F}(\mathbf{X},\mathbf{Z}) = \begin{pmatrix} \mathbf{K}^{D}(\mathbf{X},\mathbf{Z}) - \mathbf{K}^{S}(\mathbf{X},\mathbf{Z}) \\ \mathbf{L}^{D}(\mathbf{X},\mathbf{Z}) - \mathbf{L}^{S}(\mathbf{X},\mathbf{Z}) \\ \mathbf{Q}_{1}^{D}(\mathbf{X},\mathbf{Z}) - \mathbf{Q}_{1}^{S}(\mathbf{X},\mathbf{Z}) \\ \vdots \\ \mathbf{Q}_{s}^{D}(\mathbf{X},\mathbf{Z}) - \mathbf{Q}_{s}^{S}(\mathbf{X},\mathbf{Z}) \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{pmatrix}$$

Each entry denotes the market clearing condition on the whole time path. Now $\mathbf{F}(\mathbf{X},\mathbf{Z})$ has size $T \times 12$. $\mathbf{F}_{\mathbf{X}} : (T \times 12) \times (T \times 12)$, $\mathbf{F}_{\mathbf{Z}} : (T \times 12) \times (T \times 11)$. The resulting general equilibrium Jacobian matrix $\mathbf{G} \equiv -\mathbf{F}_{\mathbf{X}}^{-1}\mathbf{F}_{\mathbf{Z}}$ will have size $(T \times 12) \times (T \times 11)$.

To compute the Jacobian matrices $\mathbf{F}_{\mathbf{X}}$ and $\mathbf{F}_{\mathbf{Z}}$ efficiently, I adopted the "Fake News" algorithm developed in Auclert et al. (2021). For analytical expressions and proofs, I defer the readers to Auclert et al. (2021).

C Data Sources

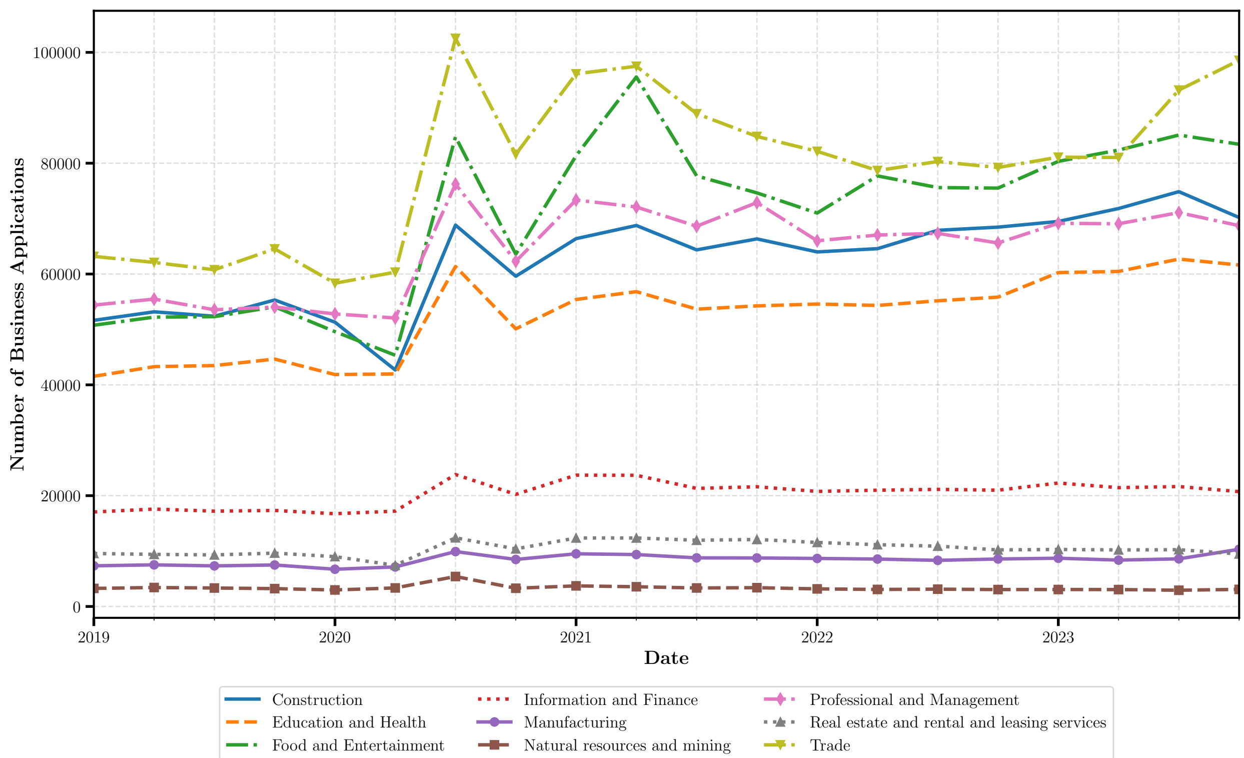
In this section, I document the surge in new business formations among the downstream sectors (Fazio et al., 2021; Decker and Haltiwanger, 2023). The most relevant data sources are Business Formation Statistics (BFS) and Business Employment Dynamics (BED).

BFS publishes monthly data on IRS Employer Identification Number (EIN) applications: All employer businesses and non-employer corporations and partnerships are required to have an EIN, while many non-employer sole proprietors also obtain one for business purposes. The total applications series, labeled "BA" in BFS files, includes all EIN applications that represent potential employer or non-employer (zero-employee) businesses. This excludes applications for trusts, estates, and financial instruments. My primary focus is on employer businesses, so I concentrate on what are termed likely employer applications ("high-propensity applications" or "HBA" in BFS files). These applications are identified by Census Bureau modeling based on characteristics with a high likelihood of transitioning into employer businesses with paid employees, such as planned hiring and corporate legal form.

BED includes quarterly data on establishment openings, closings, births, exits, expansions, and contractions, along with their related job flows. Crucially, in the BED, an establishment (or firm) birth signifies an establishment (or firm) that did not previously exist. A new firm requires a new business application, whereas a new establishment of an existing firm does not require but may acquire a new EIN.

To derive the model's counterpart in the change of total number of firms by sector, I make the

Figure 18: High Probability Business Applications By Sectors



This figure shows the numbers of business applications with high probability of becoming actual businesses by sector. Data is published quarterly by BFS. The X-axis is quarterly date. Y-axis is the number of business applications.

following assumptions regarding the data: The economy before 2020 was at steady state, with no growth in the total number of firms. This does not mean there is no firm dynamics. Firms close and open due to idiosyncratic productivity shocks to the entrepreneurs. But at sectoral level, the total number is constant. Denote the number of firm closures by C_t and the number of firm creations by N_t. Denote the total number of firms at steady state by M_{ss}. In impulse response function of firm dynamics from the model can be expressed as (for sector s, but ignoring for simplicity):

$$IRF_t = \frac{\sum_{\tau=ss+1}^{t} N_\tau - \sum_{\tau=ss+1}^{t} C_\tau}{M_{ss}} \tag{C.1}$$

With BED, N_τ and C_τ are both in principle observable. However, the cumulative net firm openings exhibit a strong upward trend component, which does not match the steady-state interpretation of pre-COVID periods in the model. To account for this, I added an unobservable sector-specific trend component K_τ in Equation (C.1)

$$IRF_t = \frac{\sum_{\tau=ss+1}^{t} N_\tau - \sum_{\tau=ss+1}^{t} C_\tau + \sum_{\tau=ss+1}^{t} K_\tau}{M_{ss}}.$$

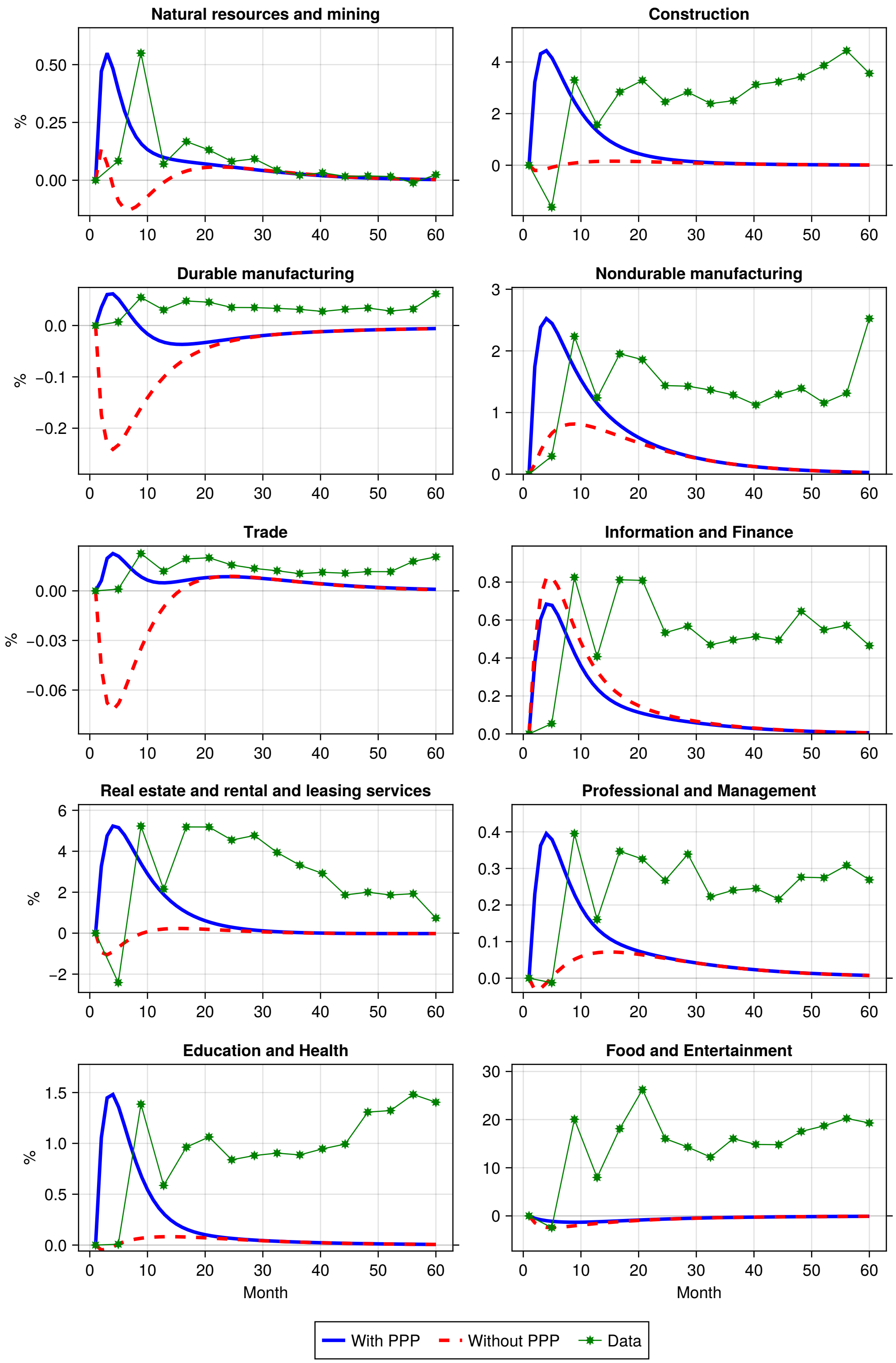
Now IRF_t can be interpreted as a series of normalized variations around the trend component, as shown by the green lines in Figure 15.

Extra work is needed if using BFS instead. Since I do not observe the number of firm closures in the data, I make the next assumption: $C_\tau = \kappa N_{\tau-1}$. This means a proportion κ of the business applications enter at τ but exit at $\tau + 1$. I allow κ to be sector specific. This assumption derives from the entrepreneurship setup in the model. Since households can switch their occupation each instance frictionlessly, it is equivalent to assume all entrepreneurs exit and re-entry the next instance if they decide to do so. The presence of κ is to account for the consistent growth of firms observed in the data, but absent in the steady state of the model. With this assumption, (C.1) can be rewritten as

$$\begin{aligned} IRF_t &= \frac{\sum_{\tau=ss+1}^{t} N_\tau - \kappa N_{\tau-1}}{M_{ss}} \\ &= \frac{N_t + \sum_{\tau=ss+1}^{t-1} (1-\kappa) N_\tau - \kappa N_{ss}}{M_{ss}} \\ &= \frac{N_t/N_{ss} + \sum_{\tau=ss+1}^{t-1} (1-\kappa) N_\tau/N_{ss} - \kappa}{M_{ss}/N_{ss}}. \end{aligned}$$

For lack of a better data source, I use the number of closures and number of births of establishments from Business Employment Dynamics (BED) to calculate sectoral specific κ using pre-COVID-19 periods.

Figure 19: IRF of Sectoral Number of Entrepreneurs after COVID-19



This figure shows the responses in sectoral numbers of entrepreneurs during COVID-19 in the model. X-axis is the number of months from period 0, when the shock happens. Y-axis is the percentage change relative to the steady-state value. Blue solid lines are the IRFs with *realized shocks*. Red dashed lines are the IRFs with *COVID-19 shocks*. Green lines are data series of the number new business applications from BFS. .